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Essais sur des questions internationales en économie de l’environnement

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RÉSUMÉ

Cette thèse est constituée de trois articles. Le premier étudie le problème de pollution globale dans un contexte environnemental incertain. Le deuxième article traite des accords internationaux sur l'environnement. Le troisième article montre comment la libéralisation du commerce peut affecter le bien-être et les taxes sur la pollution dans un monde où les pays sont hétérogènes et la pollution transfrontalière.

Dans le premier article, je considère un monde dans lequel les pays souffrent uniformément de la pollution globale. Ils font face à une menace continuelle de voir les dommages causés par cette pollution globale s'accroître subitement de façon irréversible. Je caractérise le niveau des émissions, le stock de pollution, et le niveau de bien-être actualisé en équilibres coopératif et non-coopératif. L'objectif visé est d'analyser l'impact de ce type d'incertitude sur les équilibres issus des comportements stratégiques des pays. Je trouve que cette incertitude peut avoir un effet significatif sur ces équilibres. Les pays réduisent leurs émissions pour atténuer leur exposition à cette menace. Plus la menace est grande, plus les pays ajustent leurs émissions afin de réduire le stock de pollution globale. Cependant, en dépit du fait que cette incertitude diminue le bien-être net initial, elle peut à long terme avoir un effet net positif sur le bien-être.

Le deuxième article étend la classe des modèles dynamiques standards traitant des accords internationaux sur l'environnement au cas où la durée de la période d'engagement à de tels accords est un paramètre que l'on peut varier de façon exogène. Nous y étudions les évolutions dans le temps de la taille des coalitions stables, du stock de pollution et du taux d'émissions en fonction de la durée d'engagement. Nous montrons que la longueur de la période d'engagement a un effet très significatif sur l'équilibre. Trois intervalles de durée d'engagement sont identifiés pour lesquels l'équilibre et sa dynamique diffèrent considérablement. Alors que pour des durées de la période d'engagement très longues on observe des coalitions stables constituées d'un petit nombre de pays, si ces durées sont suffisamment courtes on peut observer un niveau de coopération élevé. Les durées d'engagement entre ces deux extrêmes sont caractérisées par une relation inverse entre la durée de la période d'engagement et la taille des coalitions stables.

Ces faits portent à croire qu'il faudrait accorder une attention toute particulière au choix de la durée d'engagement lors de l'élaboration de tels accords internationaux.

Le troisième article s'inscrit dans un contexte où les activités de production des pays potentiellement hétérogènes génèrent de la pollution qui peut traverser les frontières et nuire au bien-être des pays impliqués. Dans chacun de ces pays, l'état impose des taxes sur la pollution aux firmes polluantes et des tarifs à l'importation afin de corriger cette distorsion. Ce papier a pour but d'évaluer les effets que pourrait avoir une diminution des tarifs douaniers sur la production, les taxes sur la pollution et le bien-être de ces pays. La littérature existante a étudié ce problème, mais seulement dans le cadre d'un commerce bilatéral entre pays identiques. Cet article fournit un cadre d'analyse plus réaliste dans lequel les pays ne seront pas nécessairement identiques et où le commerce pourra être multilatéral. Il devient alors possible de mettre en évidence le biais introduit en négligeant ces deux facteurs. Dans ce nouveau contexte, je montre qu'une réduction des tarifs d'importation n'augmente pas nécessairement la production ; elle peut aussi nuire au bien-être, même si la pollution est purement locale.

Mots clés : Accords internationaux sur l'environnement, Pollution globale, Commerce International, Environnement incertain, asymétrie, Risque, Jeux dynamiques, Jeux différentiels stochastiques, Marchés imparfaits

ABSTRACT

This thesis is composed of three papers. The first paper studies the problem of global pollution in the context of environmental uncertainty. The second paper has to do with international environmental agreements. The third paper shows how trade liberalization can affect welfare and pollution taxes in a world of heterogeneous countries and transboundary pollution.

In the first paper, I consider a world where countries suffer uniformly from global pollution while facing a continuous threat that the damages from this global pollution will suddenly jump to an irreversible high-damage state. I characterize the equilibrium level of emissions, the equilibrium stock of global pollution and the discounted net social welfare for both the cooperative and non-cooperative equilibria. The purpose is to analyze the impact of this type of uncertainty on the equilibrium behavior of the countries. I find that this uncertainty can have a significant effect on those equilibria. Countries reduce their emissions to mitigate their exposure to that threat. As the level of threat rises, countries adjust their emissions to lower the stock of pollutant. However, although initially this type of uncertainty has the effect of lowering the discounted net welfare, it can in the long run have a net positive effect on welfare.

The second paper extends the standard model of self-enforcing dynamic international environmental agreements by allowing the length of the period of commitment of such agreements to vary as a parameter. It analyzes the pattern of behavior of the size of stable coalitions, the stock of pollutant and the emission rate as a function of the length of the period of commitment. It is shown that the length of the period of commitment can have very significant effects on the equilibrium. Three distinct intervals for the length of the period of commitment are identified, across which the equilibrium and its dynamic behavior differ considerably. Whereas for sufficiently high values of the period of commitment only self-enforcing agreements by a small number of countries are possible, for sufficiently low such values cooperation on the part of a very high number of countries can be occur. Lengths of periods of commitment between those two thresholds are characterized by an inverse relationship between the length of commitment and the

membership size of the agreement. This suggests that considerable attention should be given to the determination of the length of such international agreements.

The third paper considers a trade situation where the production activities of potentially heterogeneous countries generate pollution which can cross borders and harm the well-being of all the countries involved. In each of those countries the policy maker levies pollution taxes on the polluting firms and a tariff on imports in order to correct that distortion. The purpose of the paper is to investigate the effect of a reduction in the tariff on equilibrium pollution taxes and welfare. The existing literature has investigated this problem for trade between two identical countries. This paper analyzes the problem in the more realistic context where countries are not necessarily identical and trade can be multilateral. It becomes possible to show what bias is introduced when those two realities are neglected. I find that a tariff reduction can actually lower output ; it can also lower welfare even if pollution is purely local.

Keywords : International environmental agreements, Global pollution, International trade, Environnemental uncertainty, asymmetry, Risk, Dynamic games, Stochastic differential games, Imperfect markets

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INTRODUCTION GÉNÉRALE

La question de la gestion de la pollution environnementale est devenue un véritable défi pour les preneurs de décision depuis quelques décennies. Aujourd'hui, l'environnement est sujet à de nombreux risques (voir par exemple IPPC, 2007) et plusieurs facteurs entravent la mise sur pied de politiques environnementales efficaces. D'une part, il faut tenir compte de la nature transfrontalière de certains polluants. Les dommages associés à certains types de pollution, comme le dioxyde de carbone et le dioxyde de souffre, peuvent affecter tous les pays indépendamment de la source de pollution. D'autre part, l'autonomie des pays ne facilite guère la tâche. Il n'existe pas d'autorité supra-nationale pouvant obliger un pays à réduire ses émissions de pollution. Tous ces faits, combinés avec une connaissance limitée quant aux dommages futurs à l'environnement, requièrent l'élaboration de stratégies pour faire un arbitrage entre les bénéfices et les coûts en situation de risque.

Cette thèse examine, dans un contexte de comportements stratégiques, différentes approches de politiques environnementales. Le premier article traite du problème de contrôle des émissions de gaz à effet de serre dans un monde souffrant de la pollution globale. Le second article porte sur les accords internationaux sur l'environnement. Le troisième article étudie les effets de la libéralisation du commerce international sur le bien-être, la production et les taxes sur la pollution, dans un monde où la pollution peut être transfrontalière et les pays non identiques.

Plusieurs données scientifiques montrent que l'accumulation des gaz à effet de serre pourrait conduire le monde vers un état catastrophique. Un tel état serait irréversible et le niveau de CO2 qui pourrait le provoquer est inconnu (voir, IPCC, 2007 et U.S. Climate Change Science Program, 2000). Dans le premier article de cette thèse, nous proposons un modèle économique de ce phénomène. Pour cela, nous considérons un monde où, en tout temps, les pays subissent le même niveau de dommage généré par leur émission de gaz à effet de serre. En tout temps, il y a un risque de voir les dommages augmenter soudainement à un niveau irréversible. L'objectif est d'évaluer l'impact de ce type de risque sur le comportement des pays.

Ce problème a été largement négligé dans la littérature sur le contrôle de la pollution. Long (1992) et Van Der Ploeg et De Zeeuw (1992) ont analysé dans un jeu différentiel le problème de pollution commune entre des pays, mais dans un monde déterministe. Ils trouvent qu'à l'état stationnaire le stock de pollution de l'équilibre non-coopératif est plus grand que celui de l'équilibre coopératif. Plus récemment, Bramoullé et Treich (2009) ont incorporé l'incertitude sur les coûts des dommages pour analyser la pollution optimale entre pollueurs. Ils trouvent que le niveau d'émission est toujours plus faible sous incertitude que dans le cas sans incertitude et que l'incertitude pourrait même améliorer le bien-être. Une limite de ce papier est son cadre d'analyse statique, qui pourrait ne pas être approprié pour des études liées à la pollution par les stock plutôt que par les flux. Nous essayons de remédier à ce problème. Nous utilisons comme modèle de base celui de Dockner et Long (1993). Ces deux auteurs ont calculé l'équilibre de Nash d'un jeu différentiel entre deux pollueurs voisins dans un monde sans incertitude. Ils montrent qu'à l'état stationnaire, le stock de pollution socialement optimal peut être soutenu par celui résultant de l'équilibre de Nash si le taux d'actualisation est suffisamment petit. Rubio et Casino (2002) plus tard ont montré que ce résultat tient seulement si le stock initial de pollution est plus grand que le stock de pollution de long terme de l'équilibre coopératif.

Dans un monde sujet à un risque de saut vers le haut des dommages environnementaux, plusieurs questions se posent. Comment les preneurs de décisions vont-ils adapter leurs stratégies en réponse à de tels risques ? Par exemple, peut-il être optimal pour les preneurs de décision de réduire leurs émissions afin de se prémunir contre un tel risque ? Comment ce type de risque affectera-t-il le bien-être social ? Ces types de problèmes seront analysés dans cet article. Nous supposerons que l'état des dommages peut être soit bas, soit haut. Quand il est bas, il y a une probabilité positive et connue qu'il passe à son état élevé à une date inconnue dans le futur. Lorsqu'il est haut, il reste dans cet état pour toujours. Nous montrons que lorsque les pays agissent de façon non-coopérative, le risque d'un saut soudain des dommages affecte leur comportement de la même façon que s'ils agissaient de façon coopérative. Ils réduisent leurs émissions afin d'atténuer leur exposition au risque, ce qui en retour réduit le stock de pollution. Bien que ce risque réduise initialement le bien-être social actualisé, il peut l'accroître sur le long terme. De façon générale, l'équilibre non-coopératif crée une distorsion en termes de qualité environnementale et de bien-être. Il résulte en un bien-être social plus faible et un stock de pollution plus grand que pour l'équilibre coopératif.

Le second article de cette thèse traite des Accords Internationaux Environnementaux (AIE). Dans plusieurs contextes, les AIEs requièrent un cadre d'étude dynamique, pour la simple raison qu'ils ont affaire à un stock de pollution et ils impliquent une interaction temporelle entre pays. Deux approches ont été adoptées dans la modélisation de tels accords. L'une consiste à supposer que les stratégies d'adhésion à l'AIE et les stratégies de pollution sont établies une fois pour toutes dès la date initiale. Chacun des signataires et des non-signataires s'engagent à mettre en oeuvre un sentier d'émissions de durée infini. Une autre approche consiste à analyser le problème dans un modèle en temps discret, en supposant que les décisions d'adhésion et de pollution sont révisées au début de chaque période. Ces deux formulations correspondent à des hypothèses extrêmes sur la durée de la période de temps où les pays sont sensés s'engager. Dans la réalité, la durée de la période d'engagement peut être un élément important de la négociation et l'équilibre résultant peut en dépendre de façon non négligeable. Intuitivement, l'on peut penser que de plus courtes périodes d'engagement peuvent favoriser de plus grandes coalitions, puisque les parties auront l'option de réviser leur décision d'adhésion et de pollution plus fréquemment après avoir observé l'état de clôture résultant de l'accord précédent. Le but de ce deuxième article est d'analyser l'effet de varier la longueur de la période d'engagement sur la taille et la stabilité de tels AIEs.

Le modèle utilisé est proche de ceux de Rubio et Casino (2005) et de Rubio et Ulph (2007). Rubio and Casino (2005) ont adapté à un contexte dynamique le concept d'AIE introduit par Barrett (1994) et Carraro and Siniscalco (1993). Ils supposent qu'à la date initiale, étant donné le stock initial de pollution, les pays jouent à un jeu à deux étapes. Dans la première étape (jeu d'adhésion), anticipant le jeu de la seconde étape, les pays décident de façon non-coopérative s'il faut ou non ratifier les accords. Dans la seconde étape (le jeu de pollution), chaque non-signataire choisit de façon non-coopérative le taux d'émission qui maximise son bénéfice net actualisé, en considérant comme donné le sentier d'émission des autres pays. Les pays signataires, pour leur part, choisissent conjointement leur sentier d'émission, agissant de façon non-coopérative contre les non-signataires dans l'optique de maximiser la somme de leurs bénéfices nets escomptés. La coalition formée dans le jeu d'adhésion ne change pas durant le jeu de pollution. Ainsi, les pays s'engagent à mettre en oeuvre leur décision d'adhésion et le sentier d'émissions résultant sur l'unique période de longueur infinie. A l'aide de simulations numériques, ils trouvent que les seules coalitions stables sont celles formées de deux pays.

Rubio et Ulph (2007) ont étendu ce papier à un modèle en temps discret à horizon infini. Au début de chaque période, étant donné le stock de pollution de la période, le jeu décrit ci-haut se joue. Les pays s'engagent à mettre en application leur décision d'adhésion ou de non-adhésion et leur stratégie respective de pollution sur la période, dont la longueur est normalisée à un comme c'est habituellement le cas dans les modèles en temps discret. Dans ce contexte, les auteurs trouvent qu'un état stationnaire du stock de pollution existe auquel correspond un état stationnaire du nombre de signataires à l'AIE. En outre, le processus de transition vers cet état stationnaire est gouverné par une relation négative entre le stock de pollution et le nombre de signataires.

Dans le deuxième article de cette thèse, nous optons pour un modèle en temps continu à horizon infini, mais nous traitons la longueur de la période d'engagement comme un paramètre pouvant prendre toute valeur strictement positive. Il devient alors possible d'étudier l'effet d'une variation exogène de la durée de la période d'engagement sur la taille des coalitions stables, sur le stock de polluant, ainsi que sur leur évolution dans le temps. A part le cas extrême à une seule période de durée d'engagement infinie, il y aura une infinité de périodes d'engagement de durée donné au début de l'horizon d'étude. Au début de chacune des périodes, chaque pays décide de ratifier les accords ou pas. Les signataires choisissent alors leur taux d'émission de la période conjointement, tandis que les non-signataires prennent cette décision unilatéralement. Nous montrons d'abord de façon analytique qu'à l'issue de tout accord, les non-signataires pollueront toujours plus que les signataires, indépendamment de la durée de la période d'engagement. Nous faisons alors usage de simulations numériques pour montrer que la durée de la période d'engagement peut avoir un impact très significatif sur l'équilibre. Deux valeurs critiques de la durée d'engagement sont identifiées. Une première valeur critique existe en dessous de laquelle le modèle génère le plus grand niveau de coopération. Au dessus de cette valeur et en dessous de la seconde, il y a une relation négative entre la durée de la période d'engagement et la taille des adhésions. Au dessus de cette seconde valeur critique, l'équilibre donne le plus petit niveau de coopération. Le cas limite à une seule période de durée d'engagement infini donne la plus petite coalition possible, ce qui génère un plus petit gain de la coopération et une trajectoire du stock de pollution plus élevée qu'avec des périodes d'engagement finies.

Le troisième article de cette thèse s'intéresse à l'idée parfois avancée que la libéralisation du commerce international pourrait nuire à l'environnement. La racine du problème est que le commerce augmente le niveau de compétition ce qui pourrait inciter les gouvernements à diluer leur politique environnementale afin de garantir des gains aux firmes nationales. Cependant, en dépit de l'hétérogénéité élevée des pays dans le monde réel, les auteurs qui se sont intéressés aux effets de la libéralisation du commerce sur les taxes sur la pollution et sur le bien-être ont travaillé sous l'hypothèse très restrictive de pays identiques. Mais nous observons fréquemment des "petits" pays échanger avec des "grands" partenaires commerciaux. Dans une telle situation, prendre en compte la structure du commerce permet de mieux caractériser l'équilibre.

L'objectif de ce troisième article est donc d'introduire une asymétrie dans un modèle de commerce international et d'analyser les effets de cette asymétrie sur les résultats d'une libéralisation multilatérale du commerce. Plus précisément, nous considérons un nombre fini de pays commerçants divisés en deux groupes. Les pays sont identiques à l'intérieur de chaque groupe, mais diffèrent d'un groupe à l'autre par le nombre de firmes dans leur industrie. Le nombre de pays dans chaque groupe est un paramètre que nous pouvons varier. L'industrie de chaque pays, par son activité de production, émet une pollution qui déborde les frontières. Cette pollution a un effet néfaste sur le bien-être des pays considérés. Dans chacun de ces pays, l'état impose des tarifs à l'importation et les taxes sur la pollution aux firmes polluantes pour corriger la distorsion générée par cette pollution globale. Nous nous intéressons à l'évaluation des effets que pourrait avoir une diminution des tarifs douaniers sur la production d'équilibre, les taxes d'équilibre sur la pollution et le bien-être social d'équilibre de ces pays.

Le cadre d'analyse du problème décrit ci-dessus est celui d'un jeu du commerce oligopolistique dans lequel le tarif à l'importation sera le même pour tous les pays. Dans une première étape, dans chaque pays, l'autorité compétente décidera unilatéralement du taux de taxe sur la pollution qui maximisera le bien-être social de son pays. Dans une seconde étape, étant donné le tarif sur les exportations et le taux de taxe sur la pollution, chaque firme décidera des quantités à produire pour le marché local et pour le marché étranger.

Plusieurs auteurs se sont intéressé au problème de pollution globale dans un contexte oligopolistique international. Parmi eux, Barrett (1994), Kennedy (1994) et Markusen (1975) ce sont penchés sur la question de savoir comment les politiques environnementales décidées dans un contexte stratégique diffèrent de celles qui sont socialement optimales. Ils trouvent que les taxes sur la pollution fixées unilatéralement ne sont pas en général socialement optimales. Mais notons que ces travaux ne considèrent qu'une situation de libre échange et ce entre pays identiques. Dans ce troisième article, nous relâchons ces deux hypothèses et nous focalisons notre analyse sur les effets de la libéralisation du commerce multilatérale sur la production d'équilibre, les taxes d'équilibre ainsi que le bien-être social d'équilibre.

Le modèle utilisé s'apparente à celui de Burguet et Sempere (2003) et à celui de Baksi et Chaudhuri (2009). Burguet et Sempere (2003) ont exploré les impacts d'une réduction uniforme du tarif sur le bien-être et sur les politiques environnementales. Ils montrent qu'une réduction bilatérale du tarif peut affecter les politiques environnementales via deux canaux. Premièrement, ils trouvent qu'une réduction bilatérale de ce tarif augmente toujours la production qui, en retour, réduit les prix et augmente les dommages marginaux de la production. Ceci incite les gouvernements à renforcer leur protection environnementale par une augmentation des taxes sur la pollution. Deuxièmement, une réduction bilatérale du tarif diminue le revenu des importations et réduit le coût des exportations, donc encourage les gouvernements à diluer leur niveau de protection environnementale. L'effet net dépend de lequel de ces deux canaux domine. Ils ont montré qu'une réduction bilatérale du tarif améliore le bien-être lorsque la politique environnementale est la taxe sur la pollution. Les limites de ce papier sont qu'il considère seulement le commerce bilatéral entre pays identiques, il suppose un monopole dans chaque pays et il considère seulement la pollution locale.

Baksi et Chaudhuri (2009) ont étendu cette analyse à un nombre arbitraire de firmes dans chacun des deux pays et ont aussi considéré plusieurs types de pollution. D'une part, ils montrent que la libéralisation du commerce augmente toujours la production dans chaque pays. Elle augmente aussi les taxes sur la pollution si la pollution est suffisamment nuisible. D'autre part, elle augmente le bien-être social lorsque la pollution est purement locale.

Dans cet article, comme dans Baksi et Chaudhuri (2009), nous considérons des degrés variés d'externalité de pollution, allant de la pollution purement locale à la pollution totalement globale. Mais notre approche est plus générale sur certains points : il y a un nombre arbitraire de pays impliqués dans le commerce ; il y a un nombre arbitraire de pays divisées en deux groupes suivant le nombre de firmes dans leur industrie ; il n'y a pas nécessairement le même nombre de pays dans chaque groupe. Nous focalisons sur l'impact de ce type d'asymétrie sur les politiques multilatérales environnementales. Pour ce faire, nous calculons l'équilibre de Nash des taxes sur la pollution, de la production d'équilibre et du bien-être d'équilibre. Nous examinons les effets d'une réduction du tarif sur ces résultats d'équilibre et nous les comparons à ceux obtenus lorsque tous les pays sont identiques. Entre autres, nous comparons les résultats de la situation dans laquelle les deux types de partenaires commerciaux coexistent sur le marché mondial à celle où tous les partenaires sont identiques.

Contrairement au scénario de pays identiques, où la libéralisation du commerce augmente toujours la production, deux nouvelles situations peuvent émerger lorsque les deux types de pays coexistent. La libéralisation du commerce peut accroître la production des pays dans un groupe et réduire celle des pays dans l'autre groupe. Elle peut aussi augmenter la production de tous les pays. Comme dans Baksi et Chaudhuri (2009), dans le cas où les pays sont identiques nous trouvons que la libéralisation du commerce augmente les taxes d'équilibre sur la pollution lorsque la pollution est suffisamment nuisible. En outre, le bien-être social est concave par rapport au tarif et la libéralisation du commerce augmente toujours le bien-être lorsque la pollution est purement locale. Cependant, en présence de l'asymétrie, ces résultats peuvent ne pas tenir, tout dépendant de l'étendu de l'asymétrie et du nombre d'acteurs impliqués dans le commerce.

CHAPITRE 1

INTERNATIONAL EMISSION STRATEGIES UNDER THE THREAT OF A SUDDEN JUMP IN THE DAMAGES

Abstract

We characterize the equilibrium level of emissions, the equilibrium stock of global pollution and the discounted net social welfare for both the cooperative and non-cooperative equilibria when the countries face the threat of a sudden irreversible jump in the global damages at an unknown date. The goal is to analyze the impact of this type of uncertainty on the equilibrium behavior of the countries. We find that it can have a significant effect on those equilibria. Countries reduce their emissions to mitigate their exposure to this threat. As the level of risk rises, countries adjust their emissions to lower the stock of pollutant. However, although initially this threat has the effect of lowering the discounted net welfare, it can in the long run have a net positive effect on welfare. The non-cooperative behavior creates a social distortion in terms of environmental quality and in terms of social welfare.

1.1 Introduction

There is scientific evidence that the accumulation of greenhouse gas could drive the world to an environmental catastrophic state. Such a catastrophic state would be irreversible and the level of CO_2 which may provoke it is uncertain. [1] This type of global environmental catastrophic risk has become of special concern in recent years. Our limited subjective knowledge about such future environmental damages raises the necessity to contemplate strategies to mitigate the cost of such risks.

This issue has been largely neglected in the literature on the control of pollution. Long (1992) and Van Der Ploeg and De Zeeuw (1992) have analyzed in a differential game the common pollution problem between countries, but in a deterministic setting.

1. See for example IPCC (2007) and U.S. Climate Change Science Program (2009).

They find that, in the non-cooperative equilibrium steady state, the level of pollution is greater than in the cooperative equilibrium state state. Recently, Bramoullé and Treich (2009) have incorporated uncertain damage costs to investigate the optimal pollution control between polluters. They find that emissions are always lower under uncertainty than under certainty and that uncertainty may actually improve social welfare. A drawback of that paper is that it makes use of a static framework, which may not be appropriate to deal with stock pollution.

We try to remedy this. The basic model used in this paper is related to that of Dockner and Long (1993). They derive the Nash equilibrium in emissions of a differential pollution game between two neighboring polluters under perfect certainty. They find that the first-best steady state can be supported in the long run as a steady-state of the non-linear Nash equilibrium if the discount rate is sufficiently small. Rubio and Casino (2002) later show that this result holds only if the initial stock of pollutant lies above the steady-state level of the cooperative equilibrium.

In a world where there is a risk of disruption of future environmental damages, the question arises as to how best decision makers can adapt their strategies in response to such a risk. More precisely, can it be optimal for decision makers to reduce their emissions in order to ensure themselves again such risk ? How will such a risk impact social welfare ? Those are the type of issues addressed in this paper.

We will assume that the state of damages can be either low or high. When it is currently low, there is a positive known probability that it will jump up to its high level at some unknown future date. When it is currently high, it will stay into that state forever. We show that when countries act non-cooperatively the risk of a sudden jump in the damages impacts their behavior in the same way as it does in the cooperative equilibrium. [2] They reduce their emissions to mitigate their exposure to risk, which in turn lowers the stock of pollutant. That risk initially lowers the discounted net welfare, but, in the long-run, it can increase it. Overall the non-cooperative equilibrium creates a social distortion in terms of the environmental quality and social welfare. It results in a lower social wel-

2. Threats of disruption in resource economics have been analyzed and discussed by a few authors. See for example Loury (1983), Bergström et al. (1985), Hillman and Long (1983), Gaudet and Lasserre (2011), Long (1975) and Bahel (2011).

fare, higher emissions and a higher stock of pollutant as compared to the cooperative equilibrium.

The remainder of this paper proceeds as follows. Section 3.2 presents the model. In Section 1.3, we derive the equilibrium that results from the full coordination of emissions control. In addition, we investigate the effects of the threat of a sudden jump in the damages on that equilibrium. The Nash equilibrium of emissions is derived in Section 1.4. We then analyze the effects of the threat of a jump on the equilibrium emission levels, the equilibrium stock of pollutant and the equilibrium discounted net welfare. We also compare the outcome resulting from that analysis to those obtained from the first best. Section 1.5 concludes.

1.2 Set up of the model

Consider a world of N identical countries whose production activity has as by-product some pollution that damages a shared environmental resource. It will be assumed that one unit of production generates one unit of emission. Let q_i denote the emissions (production) of country i. The current aggregate emissions of the world is then $Q = \sum_{i=1}^{N} q_i$.

The current stock of pollutant is denoted $z(t)$. We assume that the quantity of pollutants emitted today by the world adds to the current stock of pollutant according to the following differential equation :

$$\dot{z}(t) = Q(t) - \rho z(t), \quad \rho \in (0,1) \quad z(0) = z_0, \tag{1.1}$$

where ρ is the purification rate of the stock of pollutant.

The typical country's instantaneous benefit function is given by

$$U(q(t)) = \sigma q(t) - \frac{1}{2} q^2(t),$$

where σ is a positif parameter. The stock of pollutant at each date generates the same level of damages in each country (i.e. the pollution is global). Those damages are subject to uncertainty. The damage function can be written as the product of two functions : a deterministic part, which will be assumed quadratic and denoted $D(z(t)) = \frac{1}{2}z(t)^2$; a stochastic part, denoted $\theta(t)$, which captures the stochastic state of nature. Hence, at any time t, the global damages are given by :

$$\frac{\theta(t)}{2}z(t)^2.$$

There are two states of nature : $\theta > 0$ and $\theta + m$. The state θ corresponds to low damages, whereas the state $\theta + m$ corresponds to high damages. Initially the countries are not fully informed about future realizations of the states of damages. They know however that they are two states of nature and they know the probabilities associated to them. The transition between the two states is defined by the following stochastic process :

$$\theta(t+dt) - \theta(t) = \begin{cases} \left.\begin{array}{ll} m & \text{with probability } \beta dt \\ 0 & \text{with probability } 1 - \beta dt \end{array}\right\}, & \text{if } \theta(t) = \theta \\ 0 \quad \text{with probability } 1 \ , & \text{if } \theta(t) = \theta + m \end{cases} \tag{1.2}$$

where $0 \leq \beta dt \leq 1$ and where β is a known non-negative parameter. Hence as long as the current state of nature is low damages ($\theta(t) = \theta$), with probability βdt it will jump up to the state of high damages $\theta + m$ over the interval $[t, t + dt]$. Once the state of high damages ($\theta(t) = \theta + m$) has occurred, it will never revert back to the low-damage state. The level of severity of the jump in the damages is captured by the parameter non-negative real number m. We assume that initially the state of nature is that of low damages.

The flow of net benefits to the typical country is therefore stochastic and given by ;

$$\pi(q(t), z(t), t) = \sigma q(t) - \frac{1}{2}q^2(t) - \frac{\theta(t)}{2}z^2(t). \tag{1.3}$$

For $m > 0$, the net benefit (2.2) will be discontinuous at the eventual date of the jump in the state of damages. The particular case of $m=0$ (or $\beta=0$) corresponds to the well known deterministic model of pollution control, the value of the jump being zero (or the probability of the jump occurring being zero).

To characterize the effect of the possibility of such a jump in the damages on strategic behavior of countries, we investigate, in order, the cooperative equilibrium and the non-cooperative equilibrium.

1.3 Cooperative equilibrium

In the cooperative setting, at any date t, countries decide jointly the emission levels that maximize the sum of their expected discounted net benefit. If $z(t)$ is the current stock of pollutant and $\theta(t)$ is the current state of nature, then the value function in current value at the date t is : [3]

$$W(z(t),\theta(t)) = \max_{q_1,\ldots,q_N} \{\sum_{i=1}^{N} E_t \int_t^{\infty} e^{-r(s-t)}[\sigma q_i(s) - \frac{1}{2}q_i^2(s) - \frac{\theta(s)}{2}z^2(s)]ds\},$$

subject to (2.1)-(1.2).

The Hamilton-Jaccobi-Bellman equation associated to this stochastic optimization is

$$rW(z,\theta(t)) = \max_{q_1,\ldots,q_N} \{\sum_{i=1}^{N}(\sigma q_i - \frac{1}{2}q_i^2) - N\theta(t)z^2/2 + (\sum_{k=1}^{N} q_k \quad -\rho z)W_z(z,\theta(t)) + \mathbb{E}\{\Delta W|\theta(t)\}\}, \quad (1.4)$$

where r is the discount rate and where : [4]

$$\mathbb{E}\{\Delta W|\theta(t)\} = \begin{cases} \beta[W(z,\theta+m) - W(z,\theta)], & \text{if } \theta(t)=\theta \\ 0, & \text{if } \theta(t)=\theta+m. \end{cases} \quad (1.5)$$

The first-order conditions for the maximization of the right-hand side of (1.4) are, for

3. Since the problem is autonomous and has an infinite horizon, $W(z(t),\theta(t))$ depends only on the current state variables and not explicitly on the current date t (see Kamien and Schwartz, 1981, p. 164).

4. We use the generalized Itô lemma for jump process in deriving this Bellman equation.

$i = 1, ..., N$:

$$\sigma - q_i + W_z(z, \theta(t)) \leq 0; \quad q_i \geq 0; \quad (\sigma - q_i + W_z(z, \theta(t))q_i = 0. \tag{1.6}$$

Thus, given the current state of nature, if the equilibrium emissions are positive at the date t the marginal benefit derived from polluting by a country must be equal to the marginal social cost of pollution, $-W_z(z, \theta(t))$.

For an interior solution we get, from (1.6), that :

$$q_i = \sigma + W_z(z, \theta(t)). \tag{1.7}$$

Substituting for this expression into (1.4) results in the following state dependent differential equation :

$$rW(z, \theta(t)) = N\sigma^2/2 - N\theta(t)z^2/2 + (N\sigma - \rho z)W_z + NW_z(z, \theta(t))^2/2 + \mathbb{E}\{\Delta W | \theta(t)\}. \tag{1.8}$$

Given the particular structure of (1.8), it is helpful to determine first its solution for the state of high damages and then use that solution to solve for the state of low damages.

1.3.1 The cooperative policy : state of high damages

Under this state, from (1.5), we have $\mathbb{E}\{\Delta W | \theta(t)\} = 0$. Hence (1.8) becomes :

$$rW(z, \theta + m) = N\sigma^2/2 - N(\theta + m)z^2/2 + (N\sigma - \rho z)W_z(z, \theta + m) + NW_z(z, \theta + m)^2/2. \tag{1.9}$$

It is shown in Appendix I.1 that the following value function solves (1.9) : [5]

$$W(z, \theta + m) = -\frac{A}{2}z^2 - Bz + C, \tag{1.10}$$

5. This functional form implicitly means that we restrict our attention only on linear strategies in this study. Since we have to do with a linear quadratic game, non-linear strategies may exist as well ; see among others, Tsutsui and Mino (1990) or Dockner and Long (1993). However, it can be shown that linear and non-linear strategies yield the same steady-state.

where the coefficients A, B and C are given by :

$$\begin{aligned}
&A = [-(2\rho + r) + \sqrt{(2\rho + r)^2 + 4N^2(m+\theta)}]/2N > 0,\\
&B = \sigma AN/[NA + r + \rho] > 0,\\
&C = [\sigma^2 N - 2\sigma BN + B^2 N]/2r > 0,\\
&A' \equiv \frac{\partial A}{\partial m} > 0;\ \ B' \equiv \frac{\partial B}{\partial m} > 0;\ \ C' \equiv \frac{\partial C}{\partial m} < 0 \ \text{ and }\ \sigma > B.
\end{aligned}$$

Therefore expression (1.7) states that if the state of high damages prevails at a given date when the current stock of pollutant is z, then the equilibrium emission rate will be given by :

$$q_i(z, \theta + m) = \sigma - B - Az. \tag{1.11}$$

Equation (1.11) gives the typical country's decision rule in the cooperative equilibrium once the state of high damages has occurred.

1.3.2 The cooperative policy : state of low damages

Under this state, from (1.5), we have that $\mathbb{E}\{\Delta W|\theta(t)\} = \beta[W(z, \theta + m) - W(z, \theta)]$. Substituting into (1.8) and rearranging we find that $W(z, \theta)$ is the solution of the following differential equation :

$$(r+\beta)W(z,\theta) = N\sigma^2/2 - N\theta z^2/2 + (N\sigma - \rho z)W_z(z,\theta) + NW_z(z,\theta)^2/2 + \beta W(z,\theta+m), \tag{1.12}$$

where $W(z, \theta + m)$ is given by (1.10).

Again, it is shown in Appendix I.1 that the following quadratic form provides a solution :

$$W(z, \theta) = -\frac{a_1}{2}z^2 - a_2 z + a_3. \tag{1.13}$$

where the coefficients a_1, a_2, a_3 are given by :

$$\begin{aligned}
a_1 &= [-(2\rho+r+\beta)+\sqrt{(2\rho+r+\beta)^2+4N(A\beta+N\theta)}]/2N,\\
a_2 &= \frac{N\sigma a_1+\beta B}{Na_1+\rho+r+\beta},\\
a_3 &= [\sigma^2 N+2C\beta-2\sigma N a_2+Na_2^2]/2(r+\beta),\\
a_1' &\equiv \frac{\partial a_1}{\partial \beta}>0;\ a_2' \equiv \frac{\partial a_2}{\partial \beta}>0;\ a_3' \equiv \frac{\partial a_3}{\partial \beta}<0 \text{ and } \sigma > a_2.
\end{aligned}$$

Making use of (1.7), we get that the typical country's decision rule at any date t when the state of damages is low is given :

$$q_i(z,\theta) = \sigma - a_2 - a_1 z.$$

From now on we will denote by $\nu > 0$ the uncertain date at which the jump in the damages occurs. The case of $\nu = \infty$ would correspond to a situation where the state of high damages fails to occur. We will show later on that ν is finite.

Using (2.1), the dynamic of the stock of pollutant on the interval of time $[0,\nu]$ can be rewritten as :

$$\dot{z}(t) \equiv Nq_i(z(t),\theta) - \rho z(t) = N[\sigma - a_2 - a_1 z(t)] - \rho z(t).$$

A particular solution of that differential equation is :

$$z^L = N(\sigma - a_2)/(\rho + Na_1),$$

where L stands for the state of low damages. The general solution of the homogenous equation associated to that differential equation is :

$$z(t) = \xi e^{-(\rho+Na_1)t},$$

where ξ is an arbitrary parameter. Hence the general solution of the above equation is :

$$z_L^c(t) = z^L + \xi e^{-(\rho+Na_1)t}.$$

Since $z_L^c(0) = z_0$, we have $\xi = z_0 - z^L$. Denote by $z_L^c(t)$ the equilibrium stock of pollutant, $q_L^c(t)$ the equilibrium emissions rate and $W_L^c(t)$ the discounted net welfare at date $t \in [0,\nu)$, where the superscript c stands for the cooperative equilibrium. Their respective expressions are :

$$z_L^c(t) = N(\sigma - a_2)/(\rho + Na_1) + [z_0 - N(\sigma - a_2)/(\rho + Na_1)]e^{-(\rho+Na_1)t}, \tag{1.14a}$$

$$q_L^c(t) = \sigma - a_2 - a_1 z_L^c(t), \tag{1.14b}$$

$$W_L^c(t) = -a_1 z_L^c(t)^2/2 - a_2 z_L^c(t) + a_3, \tag{1.14c}$$

Let $z_\nu^c \equiv z_L^c(\nu)$.

Using (2.1) and (1.11), we get the dynamics of the stock of pollutant after the eventual jump in the damages :

$$\dot{z}(t) \equiv q_i(z(t), \theta + m) - \rho z(t) = N[\sigma - B - Az(t)] - \rho z(t).$$

We use a similar approach as for the state of low damages to solve this differential equation. The solution of that equation with the initial condition $z(\nu) = z_\nu^c$ will be the equilibrium stock of pollutant under the high damage state once has occurred. Its expression at any date $t \in [\nu, \infty)$ is :

$$z_H^c(t) = N(\sigma - B)/(\rho + NA) + [z_\nu^c - N(\sigma - B)/(\rho + NA)]e^{-(\rho+NA)(t-\nu)}, \tag{1.15a}$$

from which we derive the equilibrium emission levels ($q_H^c(t)$) and discounted net welfare ($W_H^c(t)$) at any date $t \in [\nu, \infty)$:

$$q_H^c(t) = \sigma - B - Az_H^c(t), \tag{1.15b}$$

$$W_H^c(t) = -Az_H^c(t)^2/2 - Bz_H^c(t) + C. \tag{1.15c}$$

The following proposition characterizes the cooperative equilibrium steady state.

Proposition 1. *In the fully cooperative equilibrium, we have :*
(i) The state of high damages must occur at a finite date (i.e. $0 < \nu < \infty$*).*
(ii) The steady state of the stock of pollutant exists and its expression is given by :

$$z_c^{stea} = N(\sigma - B)/(\rho + NA).$$

The stock of pollutant converges to z_c^{stea} *and the emissions rate approaches asymptotically its steady-state,* $q_c^{stea} = \sigma - B - Az_c^{stea}$*.*
(iii) The steady state of emissions and that of the stock of pollutant are lower than they would be in the absence of the threat of a jump in the damages.

Proof. (i) Let $\ell(s) \equiv pr(\theta(t+s) = \theta|\theta(t) = \theta)$, the probability that the low damage state will occur at $t+s$ if it occurs at t.[6] From the expression for $\theta(t)$ defined by (1.2), we know that $\theta(t+s+ds) = \theta + m$ with probability $\lambda(z(t+s))ds$ if $\theta(t+s) = \theta$; $\theta(t+s+ds) = \theta$ with probability $1 - \lambda(z(t+s))ds$ if $\theta(t+s) = \theta$ and $\theta(t+s+ds) = \theta$ with probability 0 if $\theta(t+s) = \theta + m$. Therefore, we have the following :

$$\ell(s+ds) = (1 - \lambda(z(t+s))ds)\ell(s) + 0 \times (1 - \ell(s)).$$

Hence :

$$\frac{d\ell}{ds}(s) = -\beta\ell(s) \text{ for all } s \geq 0. \tag{1.16}$$

The general solution of the above differential equation is

$$\ell(s) = ce^{-\beta s} \text{ for all } s \geq 0,$$

where c is an arbitrary constant. Since $\ell(0) = 1$, we must have $c = 1$, and hence :

$$\ell(s) = e^{-\beta s} \text{ for all } s \geq 0. \tag{1.17}$$

6. The expression of $\ell(s)$ does not depend on t because the stochastic process (1.2) is time stationary.

It follows that the probability that the jump in the damage function never occurs (i.e. $\nu = \infty$) is $\ell(\infty) = 0$. Therefore $pr(0 < \nu < \infty) = 1 - pr(\nu = \infty) = 1$.
(ii) Since the lifetime of the state of low damages is finite, the dynamics of the long run of the stock of pollutant and emissions rate are given respectively by $(1.15a)$ and (1.15b). They clearly converge respectively to z_c^{stea} and q_c^{stea}.
(iii) Notice that the steady state of the stock of pollutant and the steady state of emissions in the no-uncertainty context are given respectively by $z_c^{stea}|_{m=0}$ and $q_c^{stea}|_{m=0}$. Making use of the values of A and B given in Section 1.3.1, it is easy to see that $\partial z_c^{stea}/\partial m < 0$. Thus $z_c^{stea} < z_c^{stea}|_{m=0}$. We also have $q_c^{stea} = \sigma - B - AN(\sigma - B)/(\rho + NA)$. Deriving and rearranging, we get : $\partial q_c^{stea}/\partial m = -B'\rho/(\rho+NA) - \rho NA'(\sigma - B)/(\rho+NA)^2 < 0$. Hence, $q_c^{stea} < q_c^{stea}|_{m=0}$. □

1.3.3 Effects of the threat of a jump

To investigate the effect of the threat of a jump in the damages on the equilibrium emissions rate, the equilibrium stock of pollutant, and the equilibrium welfare, we first make the comparison with what would occur in the absence of such a threat.

Denote respectively by $\tilde{z}_c(t)$, $\tilde{q}_c(t)$, and $\tilde{W}_c(t)$ the equilibrium stock of pollutant, the emissions rate and the discounted net welfare at the date t for the case of no uncertainty, which corresponds in this model to either $m = 0$ or $\beta = 0$. Then, (a) for all $t \in [0, \nu)$, $\tilde{z}_c(t) \equiv z_L^c(t)|_{\beta=0}$, $\tilde{q}_c(t) \equiv q_L^c(t)|_{\beta=0}$, and $\tilde{W}_c(t) \equiv W_L^c(t)|_{\beta=0}$; (b) for all $t \in [\nu, \infty)$, $\tilde{z}_c(t) \equiv z_H^c(t)|_{m=0}$ and $\tilde{q}_c(t) \equiv q_H^c(t)|_{m=0}$, and $\tilde{W}_c(t) \equiv W_H^c(t)|_{m=0}$.

Proposition 2. *In the cooperative equilibrium, we have :*
(i) $\tilde{z}_c(t) > z_L^c(t)$ *for all* $t \in (0, \nu]$,
(ii) $\tilde{q}_c(t) > q_L^c(t)$ *for all* $t \in [0, \nu)$,
(iii) $\tilde{W}_c(0) > W_L^c(0)$.

Proof. See Appendix I.1.1. □

Proposition 2 states that for any feasible value of the initial stock of pollutant, the emission level and its resulting stock of pollutant are lower under the threat of a jump in the damages than its absence. This holds for the whole duration of the state of low damages. The following corollary shows that those results also hold in the state of high damages.

Corollary 1. *In the cooperative equilibrium, after the failure of the state of low damages, namely during the interval of time* $[\nu,\infty)$*, the following results hold.*
(i) $\tilde{z}_c(t) > z_H^c(t)$ *for all* $t \in [\nu,\infty)$*.*
(ii) $\tilde{q}_c(t) > q_H^c(t)$ *for all* $t \in [\nu,\infty)$*.*

Proof. See Appendix I.1.2. □

Thus the countries anticipate the fact that, although the date of the jump in damages is uncertain, it will occur in finite time with certainty. This incites them to alleviate their exposure to high damages by adopting a lower emissions path, which in turn generates a lower stock of pollutant.

Let us now consider the effect of an increase in the risk of an upward jump in the damages in the cooperative equilibrium. Denote by X_β the random variable representing the duration of the state of low damages and notice that $pr(X_\beta > s) = pr(\theta(s) = \theta|\theta(0) = \theta) = \ell(s) = e^{-\beta s}$. Hence, if $\beta_1 > \beta_2 \geq 0$, then $pr(X_{\beta_2} > s) > pr(X_{\beta_1} > s)$ for all $s > 0$ and $pr(X_{\beta_2} > 0) = pr(X_{\beta_1} > 0) = 1$.[7] Since $\beta_1 > \beta_2 \geq 0$ are arbitrary, we can conclude that at each date $t \in [0,\nu)$, the risk of the jump occurring in the next instant is increasing in β. We have the following results.

Proposition 3. *In the cooperative equilibrium,*
(i) An exogenous increase in the risk of an upward jump in the damages lowers the current stock of pollutant at all positive dates.
(ii) At the initial date, an increase in the risk of an upward jump in the damages always decreases the discounted net welfare. In the long run, such an increase has no effect on the discounted net welfare.

Proof. (i) It was shown in Proposition 2 that $\partial z_L^c(t)/\partial\beta < 0$ for all $t \in (0,\nu]$. Since $z_L^c(\nu) = z_\nu^c$, we then have $\partial z_\nu^c/\partial\beta < 0$. From (1.15a), we can derive the following :

7. Those inequalities also imply that X_{β_2} first-order stochastically dominates X_{β_1}. For more details, see for example Gollier (2004).

$\partial z_H^c(t)/\partial\beta = (\partial z_\nu^c/\partial\beta)e^{-(\rho+NA)(t-\nu)} < 0$ for all $t \geq \nu$.
(ii) Since $z_0 \geq 0$, $a_1' > 0$, $a_2' > 0$ and $a_3' < 0$, it is an easy matter to derive from (1.14c) the following $\partial W_L^c(z_0)/\partial\beta = -a_1' z_0^2/2 - a_2' z_0 + a_3' < 0$. In the long run the discounted net welfare is equal to : $W_H^c(z_c^{stea}) = -A(z_c^{stea})^2/2 - Bz_c^{stea} + C$. It does not depend on β, the sole parameter that allows us to capture variations in risk. □

At any date, the comparison between the discounted net welfare under the threat of a jump, $W(z(t))$, and that with no risk of a jump, $\tilde{W}(\tilde{z}(t))$, can be carried out as follows :

$$W(z(t)) - \tilde{W}(\tilde{z}(t)) = \{W(z(t)) - W(\tilde{z}(t))\} + \{W(\tilde{z}(t)) - \tilde{W}(\tilde{z}(t))\}.$$

The threat lowers the stock of pollutant, keeping the risk fixed (first term). The same current stock of pollutant generates more risk (second term). The first term on the right-hand side captures the strategic effect. Since $W''(z) < 0$ and $W'(z) < 0$, this effect is always positive except at the initial date. [8] The second term on the right-hand side captures the effect of the threat of a jump in the damages, which by Proposition 2 is always negative. Both effects work in opposite directions, so that the net effect can be either positive or negative.

In the long run, it is possible for the discounted net welfare under the threat to be greater than in the no-threat case. Indeed, there exist values of the parameters for which this is the case. For instance, with $\sigma = 100$; $\rho = 0.005$; $r = 0.025$; $\theta = 1$; $N = 2$; $\beta = 1$; $m = 100$, we get $W(z_c^{stea}) = -0.594 > -59.995 = \tilde{W}(\tilde{z}_c^{stea})$. Therefore the existence of the threat can improve social welfare. More generally, we show in Appendix I.1.3 that, in the long run, the strategic effect can dominate the threat effect, but only if $\rho(r+\rho)^2 \leq N^2\theta(r-\rho)$.

It is interesting to note that some results differ from those obtained by Bramoullé and Treich (2009) in their static model. They analyze the effects of uncertainty on the optimal emissions and welfare for risk-averse polluters. One of their results is that with a constant-elasticity damage function, small risks would have a net positive effect on

8. At the initial date, since $z(0) = \tilde{z}(0) = z_0$, the expression of the strategic effect is : $W_L^c(z_0) - W_L^c(z_0)$, which is equal to zero so that the uncertainty effect outweighs the strategic one.

welfare. In this paper the damage function has a constant elasticity with respect to the stock of pollutant, but polluters are risk-neutral. We have shown that uncertainty lowers the discounted net welfare at the initial date, irrespective of the level of risk.

1.4 Non-cooperative equilibrium

This section derives the Nash equilibrium for the differential game in pollution control defined by (2.1), (1.2) and (2.2). In this setting, at any date t, each country decides unilaterally the emission strategy that maximizes its own discounted net benefit, considering as given the emission strategies of the other countries. The countries being identical, we restrict attention to symmetric equilibria. If $z(t)$ is the current stock of pollutant and $\theta(t)$ the current state of nature, the current value function of the typical country j, $j = 1, ..., N$, is :

$$V(z, \theta(t)) = \max_{q_j}\{E_t \int_t^{\infty} e^{-r(s-t)}[\sigma q_j(s) - \frac{1}{2}q_j^2(s) - \frac{\theta(s)}{2}z(s)^2]ds\},$$

subject to (2.1), (1.2) and (2.2).

The associated Hamilton-Jacobi-Bellman equation is :

$$rV(z, \theta(t)) = \max_{q_j}\{\sigma q_j - \frac{1}{2}q_j^2 - \theta(t)z^2/2 + (\sum_{k=1}^{N} q_k - \rho z)V_z(z, \theta(t)) + \mathbb{E}\{\Delta V|\theta(t)\}\}, \tag{1.18}$$

where :

$$\mathbb{E}\{\Delta V|\theta(t)\} = \begin{cases} \beta[V(z, \theta + m) - V(z, \theta)], & \text{if } \theta(t) = \theta \\ 0, & \text{if } \theta(t) = \theta + m. \end{cases} \tag{1.19}$$

The first-order conditions for the maximization of the right-hand side of (1.18) require, for $j = 1, ..., N$:

$$\sigma - q_j + V_z(z, \theta(t)) \leq 0; \quad q_j \geq 0; \quad (\sigma - q_j + V_z(z, \theta(t)))q_j = 0. \tag{1.20}$$

In the above expressions, $-V_z(z, \theta(t))$ represents the private marginal cost of pollution. Hence, if at date t the emissions rate of country j is positive, it must be the case that the marginal benefit derived from polluting is equal to its marginal private cost of the polluting. For such an interior solution, we have :

$$q_j = \sigma + V_z(z, \theta(t)). \tag{1.21}$$

Substituting the optimal emissions (1.21) into (1.18) yields :

$$rV(z,\theta(t)) = (N-1/2)V_z(z,\theta(t))^2 + (N\sigma - \rho z)V_z(z,\theta(t)) + \sigma^2/2 - \theta(t)z^2/2 \\ + \mathbb{E}\{\Delta V|\theta(t)\}. \tag{1.22}$$

As for the cooperative case, we will first solve (1.22) for the state of high damages before solving it for the state of low damages.

1.4.1 The unilateral policy : state of high damages

In the state of high damages (1.19) yields $\mathbb{E}\{\Delta V|\theta(t)\} = 0$. Equation (1.22) can therefore be rewritten as :

$$rV(z,\theta+m) = (N-1/2)V_z(z,\theta+m)^2 + (N\sigma - \rho z)V_z(z,\theta+m) + \sigma^2/2 - (\theta+m)z^2/2. \tag{1.23}$$

As shown in Appendix I.2, a solution is :

$$V(z,\theta+m) = -\frac{\hat{A}}{2}z^2 - \hat{B}z + \hat{C}, \tag{1.24}$$

where the coefficients $\hat{A}$, $\hat{B}$ and $\hat{C}$ are given by :

$$\hat{A} = [-(2\rho+r) + \sqrt{(2\rho+r)^2 + 4(2N-1)(\theta+m)}]/2(2N-1),$$
$$\hat{B} = \sigma N\hat{A}/[r+\rho+(2N-1)\hat{A}],$$
$$\hat{C} = [\sigma^2 - 2\sigma N\hat{B} + (2N-1)\hat{B}^2]/2r.$$

Using (1.21), we can then derive the typical country's decision rule for emission, which is given by :

$$q_j^n(z,\theta+m) = \sigma - \hat{B} - \hat{A}z. \tag{1.25}$$

1.4.2 The unilateral policy : state of low damages

In the low-damages state, $\mathbb{E}\{\Delta V|\theta(t)\} = \beta[V(z,\theta+m) - V(z,\theta)]$. Substituting into (1.22) and rearranging yields the following differential equation :

$$(N-1/2)V_z(z,\theta)^2 + (N\sigma - \rho z)V_z(z,\theta) - (r+\beta)V(z,\theta) + \beta V(z,\theta+m) + \sigma^2/2 - \theta z^2/2 = 0, \tag{1.26}$$

where $V(z,\theta+m)$ is given by (1.24).

It is shown in Appendix I.2 that the following is a solution :

$$V(z,\theta) = -\frac{1}{2}u_1 z^2 - u_2 z + u_3,$$

where :

$$\begin{aligned}
u_1 &= [-(2\rho+r+\beta) + \sqrt{(2\rho+r+\beta)^2 + 4(2N-1)(\hat{A}\beta+\theta)}]/2(2N-1), \\
u_2 &= \frac{N\sigma u_1 + \beta\hat{B}}{(2N-1)u_1 + \rho + r + \beta}, \\
u_3 &= [\sigma^2 + 2\beta\hat{C} - 2\sigma N u_2 + u_2^2(2N-1)]/2(r+\beta),
\end{aligned}$$

Letting the superscript n stand for the non-cooperative equilibrium, the decision rule for emissions is

$$q_j^n(z,\theta) = a - u_2 - u_1 z.$$

Knowledge of the positive parameters $u_1, u_2, u_3, \hat{A}, \hat{B}, \hat{C}$ allows us to summarize the characterization of the linear Markov perfect equilibrium as follows :

Proposition 4. *The N-tuple* $(q_1^n, ..., q_N^n)$ *given, for* $j = 1, 2, ..., N$, *by :*

$$q_j^n(z, \theta(t)) = \begin{cases} \sigma - u_2 - u_1 z, & \text{if } \theta(t) = \theta \\ \sigma - \hat{B} - \hat{A}z, & \text{if } \theta(t) = \theta + m \end{cases}$$

constitutes the unique stationary linear Markov perfect equilibrium and the corresponding current discounted net welfare for each country is :

$$V(z, \theta(t)) = \begin{cases} -\frac{1}{2}u_1 z^2 - u_2 z + u_3, & \text{if } \theta(t) = \theta \\ -\frac{1}{2}\hat{A}z^2 - \hat{B}z + \hat{C}, & \text{if } \theta(t) = \theta + m \end{cases}$$

It is interesting to note that for the case where $\beta = 0, m = 0$ and $N = 2$, Proposition 4 yields exactly the same linear Markov perfect equilibrium and discounted net welfare as in Dockner and Long (1993). This proposition is a generalization of their result to an arbitrary number of countries and uncertainty about the date of a possible jump in the damages.

The following proposition characterizes the non-cooperative equilibrium steady state.

Proposition 5. *In the non-cooperative emissions game,*
(i) The stock of pollutant converges asymptotically to its steady-state, $z_n^{stea} = N(\sigma - \hat{B})/(\rho + N\hat{A})$, *which is smaller than it would be in the absence of the risk of a jump in the damages and larger than it would be in the cooperative equilibrium ;*
(ii) The steady state emissions rate is $q_n^{stea} = \sigma - \hat{B} - \hat{A}z_n^{stea}$, *which is larger than the individual emissions rate in the cooperative equilibrium, but smaller than it would be if their were no risk of a jump in the damages.*

Proof. See Appendix I.2.2. □

In the non-cooperative equilibrium, the dynamics of the stock of pollutant during the state of low damages is given by :

$$\dot{z}(t) \equiv N q_j^n(z(t), \theta) - \rho z(t) = N[\sigma - u_2 - u_1 z(t)] - \rho z(t), \;\; z(0) = z_0.$$

By the same method as for the cooperative equilibrium, we derive the expressions for the stock of pollutant, the emissions rate and the discounted net welfare in the state of low damages. They are respectively given at any date $t \in [0, \nu)$ by

$$
\begin{aligned}
z_L^n(t) &= N(\sigma - u_2)/(\rho + Nu_1) + [z_0 - N(\sigma - u_2)/(\rho + Nu_1)]e^{-(\rho + Nu_1)t}, \\
q_L^n(t) &= \sigma - u_2 - u_1 z_L^n(t), \\
V_L^n(t) &= -u_1 z_L^n(t)^2/2 - u_2 z_L^n(t) + u_3.
\end{aligned}
$$

Let us set $z_L^n(\nu) \equiv z_\nu^n$. Likewise, at any date $t \in [\nu, +\infty)$ the equilibrium stock of pollutant, the equilibrium emission levels and the discounted net welfare of the state of high damages are respectively given by

$$
\begin{aligned}
z_H^n(t) &= N(\sigma - \hat{B})/(\rho + N\hat{A}) + [z_\nu^n - N(\sigma - \hat{B})/(\rho + N\hat{A})]e^{-(\rho + N\hat{A})(t-\nu)}, \\
q_H^n(t) &= \sigma - \hat{B} - \hat{A} z_H^n(t), \\
V_H^n(t) &= -\hat{A} z_H^n(t)^2/2 - \hat{B} z_H^n(t) + \hat{C}.
\end{aligned}
$$

It is shown in Appendix I.3 that the paths of emissions and of the stock of pollutant in the cooperative equilibrium are lower than those in the non-cooperative equilibrium. The reason for this is of course that the social marginal cost of pollution is higher than the private marginal cost of pollution (i.e. $-W_z(z, \theta(t)) > -V_z(z, \theta(t))$). The disincentive to pollute is therefore lower in the non-cooperative equilibrium than it is in the cooperative equilibrium. As a consequence, the non-cooperative equilibrium generates a higher stock of pollutant and a lower discounted net welfare as compared to the cooperative equilibrium. It is also shown in Appendix I.3 that the steady-state welfare is strictly lower than the steady-state welfare in the cooperative equilibrium. This is to be expected, since the non-cooperative decision rule could always have been adopted in the cooperative equilibrium, but it was not.

1.4.3 Effects of the threat of a jump

This section analyzes the effects of the threat of a jump in the damages on the equilibrium emission levels, the equilibrium stock of pollutant and the equilibrium welfare resulting from the Nash equilibrium pollution control.

Denote respectively by $\tilde{z}_n(t)$, $\tilde{q}_n(t)$ and $\tilde{V}_n(t)$ the equilibrium stock of pollutant, the emissions rate and the discounted net welfare at the date t in the case where there is no risk of a jump in the damages. Then, (a) for all $t \in [0, \nu)$, $\tilde{z}_c(t) \equiv z_L^n(t)_{|\beta=0}$, $\tilde{q}_n(t) \equiv q_L^n(t)_{|\beta=0}$, and $\tilde{V}_n(t) \equiv V_L^n(t)_{|\beta=0}$; (b) for all $t \in [\nu, \infty)$, $\tilde{z}_n(t) \equiv z_H^n(t)_{|m=0}$, $\tilde{q}_n(t) \equiv q_H^n(t)_{|m=0}$, and $\tilde{V}_n(t) \equiv V_H^n(t)_{|m=0}$.

The following proposition compares the dynamics of the equilibrium stock of pollutant, the equilibrium emission levels and the equilibrium discounted net welfare when there is a risk of a jump in the damages to that in the absence of such a risk.

Proposition 6. *In the non-cooperative emissions game, we have :*
(i) $\tilde{q}_n(t) > q_L^n(t)$ *for all* $t \in [0, \nu)$, *and* $\tilde{q}_n(t) > q_H^n(t)$ *for all* $t \geq \nu$.
(ii) $\tilde{z}_n(t) > z_L^n(t)$ *for all* $t \in (0, \nu]$, *and* $\tilde{z}_n(t) > z_H^n(t)$ *for all* $t \geq \nu$.
(iii) $\tilde{V}_n(0) > V_L^n(0)$.

Proof. See Appendix I.2.3. □

Proposition 6 shows that implementation by the countries of the non-cooperative emission control under the risk of a sudden jump in the damages will result in a lower emission path and a lower stock of pollutant path than if there were no risk of such a jump. This result is the same as in the cooperative equilibrium. The reason for this similarity is that the damages harm the countries equally and the state of high damages will occur with certainty in finite time. Damages will be severe in the state of high damages if countries were to decide not to reduce their emissions when faced with the threat of the jump in damages. Thus the risk of a sudden jump in the damages increases the incentive to cut emissions as compared with the case of no risk, regardless of whether the countries act cooperatively or non-cooperatively in controlling their emissions.

Exactly as for the cooperative equilibrium, it can also be shown that an exogenous increase in the risk of an upward jump in the damages lowers the current stock of pollutant. Moreover, at the initial date, an increase in the risk of a jump always decreases the discounted net welfare, whereas in the long run it has no effect on the discounted welfare. In the long run, using a similar reasoning as for the cooperative setting, it can be shown that this type of risk will again result in a higher welfare for each country than in its absence only if $\rho^2(\rho+2r) \leq \theta[N(r+\rho)-2\rho]$.

1.5 Conclusion

This paper has extended the model of pollution control by Dockner and Long (1993) in two respects. First, an arbitrary number of countries are involved in the pollution activity. Second, at each instant those polluters suffer from the risk of a sudden jump in their common damages. It turns out that the equilibrium outcome is affected in much the same way by the threat of a jump in damages whether the countries act cooperatively or non-cooperatively. The discounted welfare, the emissions path and the path of the stock of pollutant are lower than in the absence of the risk. An increase in this risk decreases the discounted welfare and lowers the time path of the stock of pollutant. However, in the long run, it is possible for this type of uncertainty to have a net positive effect on welfare. But, as can be expected, the non-cooperative outcome always results in a lower environmental quality and a lower welfare than the cooperative outcome.

CHAPITRE 2

THE EFFECTS OF THE LENGTH OF THE PERIOD OF COMMITMENT ON THE SIZE OF STABLE INTERNATIONAL ENVIRONMENTAL AGREEMENTS

Abstract

This paper extends the standard model of self-enforcing dynamic international environmental agreements by allowing the length of the period of commitment of such agreements to vary as a parameter. It analyzes the pattern of behavior of the size of stable coalitions, the stock of pollutant and the emission rate as a function of the length of the period of commitment. It is shown that the length of the period of commitment can have very significant effects on the equilibrium. Three distinct intervals for the length of the period of commitment are identified, across which the equilibrium and its dynamic behavior differ considerably. Whereas for sufficiently high values of the period of commitment only self-enforcing agreements by a small number of countries are possible, for sufficiently low such values cooperation on the part of a very high number of countries can be occur. Lengths of periods of commitment between those two thresholds are characterized by an inverse relationship between the length of commitment and the membership size of the agreement. This suggests that considerable attention should be given to the determination of the length of such international agreements.

2.1 Introduction

In many contexts, International Environmental Agreements (IEAs) necessarily involve dynamic considerations. This is because they have to deal with stock pollutants and involve interactions over time among countries. Two approaches have been adopted in modeling such agreements. One consists in assuming that membership and emission strategies of the signatories and non-signatories are determined once and for all at the outset, with each of the signatories and non-signatories committing to an infinite path of emissions. Another consists in analyzing the problem in a discrete-time framework and assuming that membership and emission decisions are revised every period.

Those two formulations correspond to two very particular assumptions about the length of the period of time for which the countries are required to commit. In reality the length of the period of commitment can be an important element of negotiation and the resulting equilibrium may well depend significantly on this length. Intuitively, one might think that a short period of commitment could favor a larger coalition size than a longer one, since the parties will then have the option of revising their membership and emission decisions more frequently, after having observed the state that results at the close of the previous agreement. The purpose of this paper is to analyze the effect of varying the length of the period of commitment on the size and stability of such IEAs. [1]

The model used is closely related to that of Rubio and Casino (2005) and Rubio and Ulph (2007). Rubio and Casino (2005) adapt to a dynamic framework the concept of IEA introduced by Barrett (1994a) and Carraro and Siniscalco (1993). They assume that at the initial date, given the initial stock of pollutant, countries play a two-stage game. In the first stage (the membership game), anticipating the play of the game in the second stage, the countries decide non-cooperatively whether or not to join the agreement. In the second stage (the emission game), each non-signatory decides non-cooperatively the emission rate that maximizes its discounted net benefit, taking as given the emission path of the other countries. Signatory countries choose jointly their emission path, acting non-cooperatively against non-signatories in order to maximize their aggregate discounted net benefits. Signatories also take as given the strategy of non signatories. The coalition formed in the membership game cannot change in the emission game. Hence countries commit to both their membership or non membership decision and to their respective emission paths for a period of infinite length. Using numerical simulations, they find that a two-country coalition is the only self-enforcing IEA.

Rubio and Ulph (2007) extend that paper to an infinite-horizon model in a discrete-

1. Reinganum and Stokey (1985) explore the impact of the length of the period of commitment on the optimal extraction of a common pool nonrenewable resource. They find that shrinking the length of the period of commitment leads to a quick depletion of the resource. But they do not address the issue of coalition formation in their paper.

time framework. At the outset of each period, given the stock of pollutant at the beginning of the period, the play of the game is as in the game described above. Countries commit to membership or non membership and to their respective emission strategies for the duration of the period, *whose length is normalized to one* as is usually the case in discrete-time modeling. The authors find that, in this context, there exists a steady-state stock of pollutant and a corresponding steady-state IEA membership size and that, in the transition towards this steady-state, the membership size and the stock of pollutant vary inversely.

In this paper, we adopt an infinite horizon continuous-time framework, but treat the length of the period of commitment as a parameter that can take any strictly positive value. It is thus possible to study the effect of exogenously varying the length of the period of commitment on the equilibrium size of the stable coalition and stock of pollution, as well as on their pattern of behavior over time. Except for the extreme case of a single period of commitment of infinite length, there will be an infinite number of periods of commitment, the length of which is exogenously given at the outset. At the begin of every period of commitment, each country decides whether or not to adhere to the agreement. The signatories then jointly decide on their emission rate for the period of commitment, while the non-signatories make that decision unilaterally.

It is first shown analytically that non-signatories always pollute more than signatories and that they always gain more than signatories from any agreement, irrespective of the length of the period of commitment. Numerical simulations are then used to show that the length of the period of commitment can have a very significant effect on the equilibrium. Two critical values of the length of commitment come out. A first critical value is shown to exist below which the model generates the highest level of cooperation. Above this critical value and below the next one, there is a negative relationship between membership size and the length of commitment. Above this second critical value, the equilibrium yields the smallest level of cooperation.

The limiting case of a single period of commitment of infinite length is shown to yield the smallest possible coalition. This generates a lower gain from cooperation and a higher trajectory of the stock of pollutant than that which arises in the case of finite lengths of commitment.

The remainder of the paper is organized as follows. Section 3.2 sets out the model. Section 3.2.1 resolves the second stage of the game. In addition, the outcomes of the cooperative and the non-cooperative equilibria are derived in that section. Section 2.4 presents the first stage of the game. In Section 2.5, the importance of the choice of the length of the period of commitment is investigated by simulation. Section 3.5 concludes.

2.2 The model

Consider the formation of an infinite sequence of IEAs, in which countries can make binding commitments about their emission rates and their membership decision over a limited horizon. Define a *period* to be the interval of (continuous) time over which countries can make such commitments, and let h be the length of the period. Assume an infinite number of such periods, $[0,h],[h,2h],[2h,3h],...,$ and N identical countries, $i = 1,...,N$. Each country makes a membership decision and commits to a level of emission for each of the intervals $[0,h],[h,2h],[2h,3h],...,$. We will assume that one unit of production generates one unit of emissions. Let q_i denote the emissions of country i. Following Rubio and Ulph (2007), assume that at each instant $q_i \in [0,1]$. The current aggregate emissions of the world is then $Q = \sum_{i=1}^{N} q_i \in [0,N]$.

The current stock of pollutant is denoted $z(t)$. We assume that the amount of pollutants emitted today by the world adds to the current stock of pollutant according to the kinematic equation

$$\dot{z}(t) = Q(t) - \rho z(t), \quad \rho \in (0,1) \quad z(0) = z_0, \tag{2.1}$$

where ρ is the natural purification rate.

The stock of pollutant at each date generates damage costs for each country which we assume to be a quadratic function of the stock : $\frac{\gamma}{2}z^2$, with γ positive constant. As in Ulph (2004) and Rubio and Ulph (2007) the instantaneous benefit function is assumed to be linear in current emissions : aq, where a is a positive constant. Thus the flow of net benefits to a country is given by

$$\pi(q,z) = aq - \frac{\gamma}{2}z^2. \tag{2.2}$$

At the beginning of every period, each country determines an emission strategy for that period. A country's choice will depend on the beginning-of-period stock and the length of the period, h. Let $q_j^k(z_k)$ denote the emission strategy planned by country j for period k when the stock of pollutant at the outset of the period is $z(kh) = z_k$.

The model of IEA formation in each period is a dynamic version of the model of self-enforcing IEAs introduced by Carraro and Siniscalco (1993) and Barrett (1994a) and the continuous-time version of Rubio and Ulph (2007). At the beginning of each period, given the initial stock, there is a two-stage game. In the first stage (the membership game), countries first decide whether or not to join an IEA. In the second stage (the emission game), non-signatory countries choose their emissions for the current period non-cooperatively, while signatory countries act in a cooperative fashion.

For example, for the period $[kh,(k+1)h]$, given the initial stock of pollutant of the current period z_k, countries play the two-stage game at the initial date $t = kh$ of the current period. The membership decision which results from the membership game and the emission strategy q_j^k of a given country j are thus decided at the initial date of the current period. For simplicity we will assume that it commits to a constant q_j^k for the duration of period k.

Countries being identical, we will also assume that there is a binomial random variable whose realization at any given period determines whether a particular country will be among the members or not for the period. For any stable IEA of size $n \leq N$ in that period, the *a priori* probability of any given country being a member of the coalition is n/N. Because of the identical countries assumption, this probability is the same for all countries and is independent of the history of membership decisions of the country. Therefore each country has the same expected present value of current and future net benefits, which will depend on the initial stock of pollutant of the next period. We will denote by $\Psi(z_k)$ the expected present value of current and future net benefits of the representative country when the stock of pollutant at the outset of the period is z_k.

In each period, the second stage of the game is solved first, taking as given the set of signatories of the membership game.

2.3 The second stage of the game

Consider some beginning of period date $t \in \{0, h, 2h, 3h, 4h, ...\}$, when the stock of pollutant is $z(t)$. Let $K(S)$ denote the set of signatories and n the number of signatories at that date. The current value function of a non-signatory is then

$$V_j(n, z(t)) = \max_{q_j \in [0,1]} \left\{ \int_t^{t+h} e^{-r(s-t)} \pi(q_j, z(s)) ds + e^{-rh} \Psi(z(t+h)) \right\}, \tag{2.3}$$

subject to (2.1) and (2.2), where r is the discount rate.

The *aggregate* current value function of all signatories at the same date is

$$V_S(n, z(t)) = \max_{q_i, i \in K(S)} \{ \int_t^{t+h} e^{-r(s-t)} \sum_{i \in K(S)} \pi(q_i, z(s)) ds + n e^{-rh} \Psi(z(t+h)) \}, \tag{2.4}$$

again subject to (2.1)-(2.2) and $q_i \in [0,1]$, for all $i \in K(S)$.

The countries being identical, the value function of signatory i is $V_i(n, z(t)) = V_S(n, z(t))/n$, $\forall i \in K(S)$.

Definition 1. *In an infinite-duration game defined by (2.3) and (2.4), with the length of period h, an emission strategy for a country j is a sequence of functions $q_j \equiv \{q_j^k : [kh, (k+1)h] \times R_+ \to R_+\}_{k=0}^{\infty}$, where q_j^k is a constant function of $s \in [kh, kh+h]$, for $k = 0, 1, 2, ...$*

This means that at the outset of a given period, given the coalition formed in the membership game, each country chooses and commits to use a constant emission rate in the emission game.

This continuous-time problem can be transformed into a discrete-time one. Indeed, on a given interval $[kh, (k+1)h]$ the emission strategies of the players are constant and so is the aggregate emission of the world, Q. Hence, the solution of the differential equation

(2.1), given the initial stock of pollutant $z(kh) = z_k$, is :

$$z(t) = \frac{Q}{\rho} + (z_k - \frac{Q}{\rho})e^{-\rho(t-kh)} \quad \forall t \in [kh,(k+1)h]. \tag{2.5}$$

So, at time $t = (k+1)h$, the dynamic evolution of the stock of pollutant between the outset of periods k and $k+1$ is given by :

$$z((k+1)h) \equiv z_{k+1} = f(\rho,h)Q + z_k e^{-\rho h}, \tag{2.6}$$

where $f(x,h) = (1-e^{-hx})/x, \quad \forall x > 0$. We adopt this notation in the remainder of the paper. The following integral yields the net benefit function at each period, which depends on the length of the period :

$$\int_{kh}^{(k+1)h} e^{-r(s-kh)}\pi(q,z(s))ds = aqf(r,h) + D(Q,z_k), \tag{2.7}$$

where $D(Q,z_k) = -\frac{\gamma}{2}[(\frac{Q}{\rho})^2 f(r,h) + (z_k - \frac{Q}{\rho})^2 f(r+2\rho,h) + 2\frac{Q}{\rho}(z_k - \frac{Q}{\rho})f(r+\rho,h)]$.

Substituting (2.6) and (2.7) into (2.3), we obtain the Bellman equation for non-signatories :

$$V_j(n,z_k) = \max_{q_j\in[0,1]}\{aq_j f(r,h) + D(Q,z_k) + e^{-rh}\Psi(f(\rho,h)Q + z_k e^{-\rho h})\}. \tag{2.8}$$

Similarly, substitution of (2.6) and (2.7) into (2.4) yields the Bellman equation for all signatories :

$$V_S(n,z_k) = \max_{q_i, i\in K(S)}\{\sum_{i\in K(S)} aq_i f(r,h) + nD(Q,z_k) + ne^{-rh}\Psi(Qf(\rho,h) + z_k e^{-\rho h})\}, \tag{2.9}$$

subject to $q_i \in [0,1]$ for all $i \in K(S)$.

The best response of the countries results from the strategic behavior between signatories and non-signatories. For the sake of simplicity, we will restrict the set of parameters to be such that the dominant strategy for non-signatories at each period will be to pollute the maximum. The necessary and sufficient condition for this is :

$$af(r,h) \geq \lambda_1 Q + z_k\lambda_2 - f(\rho,h)e^{-rh}\Psi'(Qf(\rho,h)+z_k e^{-\rho h}), \tag{2.10}$$

where $\lambda_1 = \frac{\gamma}{\rho^2}(f(r,h)+f(r+2\rho,h)-2f(r+\rho,h))$ and $\lambda_2 = \frac{\gamma}{\rho}(f(r+\rho,h)-f(r+2\rho,h))$.

The Kuhn-Tucker conditions for the optimization problem (2.9) are, for $i \in K(S)$:

$$[af(r,h)-n(\lambda_1 Q+z_k\lambda_2 - f(\rho,h)e^{-rh}\Psi'(Qf(\rho,h)+z_k e^{-\rho h}))-\mu_i]q_i = 0; q_i \geq 0, \tag{2.11}$$

$$af(r,h)-n(\lambda_1 Q+z_k\lambda_2 - f(\rho,h)e^{-rh}\Psi'(Qf(\rho,h)+z_k e^{-\rho h}))-\mu_i \leq 0, \tag{2.12}$$

$$(1-q_i)\mu_i = 0;\;\; \mu_i \geq 0;\;\; 1 \geq q_i, \tag{2.13}$$

where μ_i is the Kuhn-Tucker multiplier associated to the constraint $q_i \leq 1$. So, if $q_i(n,z_k)$ satisfies simultaneously (2.10)-(2.13), the payoff for a non-signatory and that for a signatory will be respectively given by

$$V_j(n,z_k) = af(r,h)+D(Q(n,z_k),z_k)+e^{-rh}\Psi(f(\rho,h)Q(n,z_k)+z_k e^{-\rho h}), \tag{2.14}$$

$$V_i(n,z_k) = aq_i(n,z_k)f(r,h)+D(Q(n,z_k),z_k)+e^{-rh}\Psi(Q(n,z_k)f(\rho,h)+z_k e^{-\rho h}), \tag{2.15}$$

where, $Q(n,z_k) = nq_i(n,z_k)+(N-n)$.

Proposition 7. *The current emissions by signatories are always less than the current emissions by non-signatories and the resulting payoff of non-signatories is always greater than that of signatories for* $n \in [2, N-1]$.

Proof. By construction, we have $0 \leq q_i(n,z_k) \leq 1 = q_j(n,z_k)$. Using (2.14) and (2.15), since $0 \leq q_i(n,z_k) \leq 1$, we get : $V_j(n,z_k) - V_i(n,z_k) = af(r,h)(1-q_i(n,z_k)) \geq 0$. ■ □

Proposition 7 says that non-signatories pollute more than signatories and they also gain more than signatories from any agreement. These results are known in the literature.

They have been shown, among others, by Rubio and Ulph (2007) in their discrete-time model, with the length of the period of commitment set equal to one, and by Rubio and Casino (2005), with a period of commitment of infinite length. This proposition show that those results hold irrespective of the length of the period of commitment.

Before solving the first stage of the game, it is useful to study the particular cases of the non-cooperative equilibrium ($n = 0$ or $n = 1$) and the fully-cooperative equilibrium ($n = N$).

2.3.1 The non-cooperative equilibrium

Assume that all N countries decide non-cooperatively the emission strategy of the current period that maximizes their discounted net benefit, taking as given the current emission strategy of the other countries. The Bellman equation is then the special case of (2.8) for which $n = 1$ or $n = 0$ and $\Psi = V_j$, and we have the following result.

Proposition 8. *Assume that*

$$z_0 \leq \tilde{z}; \quad af(r,h) \geq N\lambda_1 + \tilde{B}e^{-rh}f(\rho,h) + \tilde{z}[\lambda_2 + \tilde{A}e^{-rh}f(\rho,h)]. \tag{2.16}$$

In the non-cooperative equilibrium each country will emit $q_j = 1$ at each period. The sequence of pollutant stocks at the outset of each period, $\{z_k\}_{k=0}^{\infty}$, increases and converges asymptotically to a steady state

$$\tilde{z} = \frac{N}{\rho}.$$

The present discounted net welfare for any country j is given by

$$V_j(z_0) = -\tilde{A}z_0^2/2 - \tilde{B}z_0 + \tilde{C},$$

where $\tilde{A} = \gamma/(r+2\rho) > 0$, $\tilde{B} = N[\lambda_2 + \tilde{A}f(\rho,h)e^{-h(r+\rho)}]/(1-e^{-h(r+\rho)}) > 0$, and $\tilde{C} = [af(r,h) - N^2\lambda_1/2 - e^{-rh}Nf(\rho,h)(\tilde{B} + \tilde{A}Nf(\rho,h))]/(1-e^{-rh})$.

Proof. For $n = 1$, first we assume that $q_j(n,z_k) = 1$, so that $Q(n,z_k) = N$. Using a quadratic guess, $\Psi = V_j = -\tilde{A}z^2/2 - \tilde{B}z + \tilde{C}$, the left-hand side and the right-hand side of (2.14) are then second degree polynomials in z. Equating their coefficient of powers of z, one gets the values of $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$. Since $z_0 \leq \tilde{z}$, the sequence $\{z_k\}_{k=0}^{\infty}$ is increasing and it converges to $\tilde{z}$ which is its lowest upper bound. Now, for a sufficient condition to have $q_j(1,z_k) = 1$ at each period, rewrite (2.10) for $\Psi = -\tilde{A}z^2/2 - \tilde{B}z + \tilde{C}$, to get

$$af(r,h) \geq Q\lambda_1 + \tilde{B}e^{-rh}f(\rho,h) + z_{k+1}\tilde{A}e^{-rh}f(\rho,h) + z_k\lambda_2.$$

In this inequality, substituting Q, z_k and z_{k+1} by their respective upper bounds N, $\tilde{z}$ and $\tilde{z}$, and rearranging, yields the second condition of (2.16). Thus if the inequalities in (2.16) hold, it will be optimal for a country to emit $q = 1$ at each period. ■ □

2.3.2 The cooperative equilibrium

Suppose now that all the countries decide cooperatively the emission strategies of the current period that maximizes their aggregate discounted net benefit. The Bellman equation is then the particular case of (2.9) for which $n = N$ and $\Psi = V_S/N = V_i$. If the following conditions hold :

$$z_0 \leq \bar{z}; af(r,h) \leq N^2[\lambda_1 + \bar{A}e^{-rh}f(\rho,h)^2] + N\bar{B}f(\rho,h)e^{-rh}, \tag{2.17}$$

it will be optimal for a country to pollute a quantity between zero and one. We have the following result.

Proposition 9. *Assume that the conditions in (2.17) hold. In the fully-cooperative equilibrium, the sequence of pollutant stocks at the outset of each period, $\{z_k\}_{k=0}^{\infty}$, converges to a steady state*

$$\bar{z} = \frac{af(r,h) - \bar{B}Ne^{-rh}f(\rho,h)}{N(\lambda_2 + \bar{A}f(\rho,h)e^{-rh}) + \rho N[\lambda_1 + \bar{A}f(\rho,h)^2e^{-rh}]}$$

if and only if

$$R_N = \frac{\lambda_1 e^{-\rho h} - \lambda_2 f(\rho,h)}{\lambda_1 + \bar{A}f(\rho,h)^2 e^{-rh}} > -1.$$

This convergence is monotone if and only if $R_N > 0$. At any period, say k, the optimal *emission strategy for a country is given by*

$$q_i(z_k) = \frac{af(r,h) - N\bar{B}f(\rho,h)e^{-rh} - z_k N[\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)}]}{N^2[\lambda_1 + \bar{A}f(\rho,h)^2 e^{-rh}]}.$$

The present discounted net welfare for a country is given by $V_i(z_0) = (-\bar{A}z_0^2/2 - \bar{B}z_0 + \bar{C})/N$*, where* $\bar{A}$*,* $\bar{B}$*and* $\bar{C}$ *are given in Appendix II.1.*

Proof. See Appendix II.1. □

Since at each period, the emission rate by a country from the non-cooperative equilibrium is always greater than that from the cooperative equilibrium, it is so for the stock of pollutant resulting from these two types of game. Thus, when the steady state of the stock of pollutant from the cooperative equilibrium exists, it is always lower than that from the non-cooperative equilibrium. [2]

2.4 The first stage of the game

To solve the membership game we use the notion of stability introduced by D'Aspremont et al. (1983).

Definition 2. *At the beginning of a period, say k, if the current stock of pollutant is* z_k*, a coalition of signatories* $K(S)$ *of size* n^* *is said to be stable, or self-enforcing, if and only if*

$$\begin{aligned} V_i(n^*, z_k) &\geq V_j(n^* - 1, z_k) \\ V_j(n^*, z_k) &\geq V_i(n^* + 1, z_k). \end{aligned}$$

The first inequality of Definition 2 is the internal stability condition. Its interpretation is that a signatory country cannot be better off by leaving the coalition, given that the

2. Notice that these results may also hold in the presence of uncertainty, risk, various types of preferences and learning. For useful discussions about this argument, see among others Dockner and Long (1993), Ulph (2004), Nkuiya (2011a, b) and Long (1992).

other countries maintain their membership decision. The second inequality is the external stability condition. It means that a non-signatory cannot be better off by joining the coalition, given that the other countries maintain their membership decision.

Notice that the equilibrium coalition size n^* depends on the length of the period of commitment h, the current stock of pollutant z_k, and the remaining parameters of the model. To alleviate notation, it will be denoted from now on by $n^*(z_k)$ instead of $n^*(z_k,h)$.[3]

As explained in the previous section, since the countries are identical, each has *a priori* the same probability $n^*(z_k)/N$ of being a signatory. Therefore the expected present value of future net benefits for each country is :

$$\begin{aligned}\Psi(z_k) &= \frac{n^*(z_k)}{N}V_i(n^*(z_k),z_k)+(1-\frac{n^*(z_k)}{N})V_j(n^*(z_k),z_k)\\ &= aQ(n^*(z_k),z_k)f(r,h)/N+D(Q(n^*(z_k),z_k),z_k)\\ &\quad +e^{-rh}\Psi(Q(n^*(z_k),z_k)f(\rho,h)+z_k e^{-\rho h}).\end{aligned} \tag{2.18}$$

Recall that the value function Ψ must sustain simultaneously conditions (2.10)-(2.13) along with (2.18).

2.4.1 Quadratic approximation of the value function Ψ

Since, $Q(n^*(z_k),z_k)$ is not linear with respect to z_k, one can see from (2.18) that a quadratic is not a plausible guess for Ψ. In Appendix II.3, we present an algorithm for deriving a quadratic approximation, $-Az^2/2-Bz+C$, $A>0$; $B>0$, of the function Ψ. This algorithm is merely an adaptation to any $h>0$ of that by Rubio and Ulph (2007) which works only for $h=1$.

3. A given value of (z_k,h) might sustain more than one stable coalition. In a such case, we will consider as self-enforcing the stable coalition with the highest size. Thus, $n^*(z_k)$, is the greatest integer for which the internal stability holds. Indeed, if $n^*(z_k)\in[1,N-1]$ is the largest internal stable coalition, then $1+n^*(z_k)$ is not internally stable, *i.e.* $V_j(n^*(z_k),z_k)>V_i(1+n^*(z_k),z_k)$. So, $n^*(z_k)$, is simultaneously externally stable and internally stable.

Let us assume that

$$af(r,h) \geq N(\lambda_1 + \lambda_2/\rho) + f(\rho,h)e^{-rh}(B + AN/\rho), \tag{2.19}$$

$$NBf(\rho,h)e^{-rh} + Nz_0[\lambda_2 + Ae^{-h(r+\rho)}f(\rho,h)] \geq af(r,h). \tag{2.20}$$

With z_k is the stock of pollutant at the outset of the period k, we compute the three critical values of n using the formulae

$$\bar{n}_\ell(z_k) = \frac{\tau(z_k,h) + (-1)^\ell \sqrt{\tau(z_k,h)^2 - 4af(r,h)(\lambda_1 + Ae^{-rh}f(\rho,h)^2)}}{2(\lambda_1 + Ae^{-rh}f(\rho,h)^2)}, \ell \in \{1,2\}, \tag{2.21}$$

$$\bar{n}_0(z_k) = af(r,h)/\tau(z_k,h), \tag{2.22}$$

where $\tau(z_k,h) = N\lambda_1 + f(\rho,h)e^{-rh}(B + ANf(\rho,h)) + z_k(\lambda_2 + Af(\rho,h)e^{-h(r+\rho)})$. Notice that when conditions (2.19) and (2.20) hold, we have : $1 \leq \bar{n}_0(z_k) < \bar{n}_1(z_k) < N < \bar{n}_2(z_k)$. The following proposition characterizes the decision rules of emissions by signatories as well as non-signatories at each period.

Proposition 10. *Assume that conditions (2.19) and (2.20) hold. The dominant strategy for non-signatories is to emit $q_j = 1$ at each period, irrespective of the coalition size and the stock of pollutant. At any period, say k, if z_k is the current stock of pollutant, the best emission strategy for a signatory in a coalition of size n for that period is given by*

$$q_i(n,z_k) = \begin{cases} 1 & \text{if } 1 \leq n \leq \bar{n}_0(z_k) \\ \frac{af(r,h) + n^2(\lambda_1 + Ae^{-rh}f(\rho,h)^2) - n\tau(z_k,h)}{n^2(\lambda_1 + Ae^{-rh}f(\rho,h)^2)} & \text{if } \bar{n}_0(z_k) \leq n \leq \bar{n}_1(z_k) \\ 0 & \text{if } \bar{n}_1(z_k) \leq n \leq N \end{cases} \tag{2.23}$$

where $\bar{n}_0(z_k)$ is given by (2.22) while $\bar{n}_1(z_k)$ and $\bar{n}_2(z_k)$ are given by (2.21).

Proof. See Appendix II.2. □

Proposition 10 shows that as the size of the coalition increases, the emission rate of members from that coalition decreases. Making use of (2.23), we get an analogous result for the global emission which decreases as the coalition size increases.

2.4.2 Dynamics of the stock and number of signatory

In this section, we explain how we use the approximation of the value function above to derive numerically the evolution of the stock of pollution and the number of signatories over time. For a fixed value of h, to derive the equilibrium we consider the values of A, B, C given by the algorithm for which inequalities (2.19) and (2.20) hold. Given z_k, we compute $n^*(z_k)$ and $q_i(n^*(z_k),z_k)$, using the steps (5) and (4) of the algorithm. We compute $z(t)$ for $t \in (kh,(k+1)h)$, making use of (2.5). Given z_k and $n^*(z_k)$, we calculate the stock of pollution at the outset of the period, $k+1$, according to :

$$z_{k+1} = f(\rho,h)(N-n^*(z_k)) + f(\rho,h)n^*(z_k)q_i(n^*(z_k),z_k) + z_k e^{-\rho h}. \tag{2.24}$$

If the steady-state of the stock of pollution exists, (2.24) shows that it must depend steady-state emissions by signatories. If the steady-state of emissions by signatories is equal to zero, then $z^{stea} = (N-n^*(z^{stea}))/\rho$. If the steady-state emissions by signatories are given by

$$q_i(n^*(z^{stea}),z^{stea}) = \frac{af(r,h) + n^*(z^{stea})^2(\lambda_1 + Ae^{-rh}f(\rho,h)^2) - n^*(z^{stea})\tau(z^{stea},h)}{n^*(z^{stea})^2(\lambda_1 + Ae^{-rh}f(\rho,h)^2)}, \tag{2.25}$$

then, solving the equation $z^{stea} = f(\rho,h)(N-n^*(z^{stea})) + f(\rho,h)n^*(z^{stea})q_i(n^*(z^{stea}),z^{stea}) + z^{stea}e^{-\rho h}$ with respect to z^{stea} yields

$$z^{stea} = \frac{af(r,h) - n^*(z^{stea})Be^{-rh}f(\rho,h)}{n^*(z^{stea})[\rho(\lambda_1 + Ae^{-rh}f(\rho,h)^2) + (\lambda_2 + Ae^{-h(r+\rho)}f(\rho,h))]}. \tag{2.26}$$

Finally, if the steady-state of the emission level by signatories is equal to one, then the steady-state of the stock of pollutant is N/ρ. This case is analogous to the long-run equilibrium for the non-cooperative equilibrium characterized by Proposition 8.

Notice that by setting $n^*(z^{stea}) = N$ in formulae (2.25)-(2.26), one gets the same results as in the cooperative equilibrium.

2.5 Numerical simulations : the effects of the length of commitment

In this section we present the outcome of the numerical analysis on a set of $N = 20$ identical countries with $a = 367$ and $z_0 = 900$. We use the same parameter values as Rubio and Casino (2005). These are $r = 0.025$; $\gamma = 0.001$; $\rho = 0.005$. The model captures certain results of Rubio and Ulph (2007) for some values of the length of commitment and yields different results for other values.

The simulations have been carried out for more than four thousand values of the length of commitment. What first appears clearly is that the current value function of signatories as well as of non-signatories increases with the number of signatories. Of greater interest is the effect of the length of commitment on the equilibrium number of signatories and on the gains from cooperation, to which we now turn.

2.5.1 The length of commitment and the size of self-enforcing coalitions

As concerns the size of self-enforcing coalitions, our simulations reveal that for all values of the parameter considered in this study, the size of the stable coalition $n^*(z_k, h)$ is the least integer greater than or equal to $n_0(z_k, h)$. The rationale is that if a member of the coalition $n^*(z_k, h)$ unilaterally decides to leave that coalition, members of the remaining coalition of size $n^*(z_k, h) - 1$ must play the same maximum level of emissions as the defector and therefore must have the same payoff. That action is not profitable because the larger is the coalition, the greater is the associated gain to its members. Moreover, we have found that the interval $[n_0(z_k, h), n_1(z_k, h)]$ contains at most one integer. Those results imply that members of a self-enforcing IEAs never emit the maximum emission, as do non-members.

Our numerical analysis also highlights two critical values of the length of commitment, which distinguish two possible cases for the dynamics of the relation between membership size and the stock of pollutant. The first case corresponds to extreme lengths of the period commitment ($h < 182.6$ or $h > 194.8$). In this case, simulations suggest two main results. First, there is a negative relation between the length of the period of commitment and the number of signatories at each period. Second, as in Rubio and Ulph

(2007), the stock of pollutant rises asymptotically to its steady state, while the coalition size decreases over time and converges after a finite number of periods to its steady-state. This is illustrated in Figure 2.1 for the case where the length of the period of commitment is equal to $h = 1$.

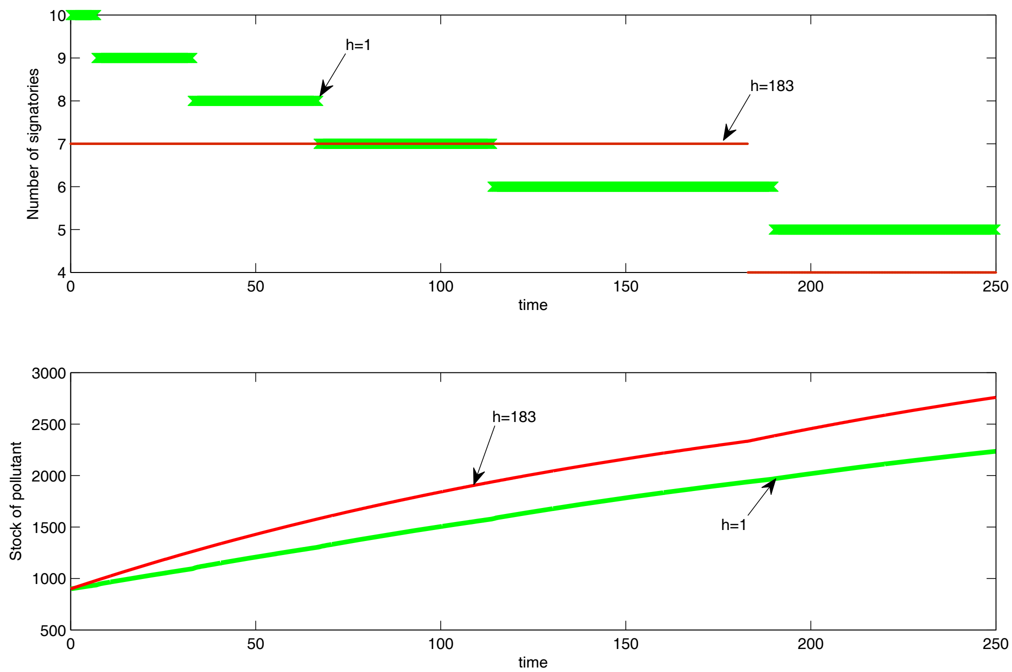

Figure 2.1 – Length of commitment, coalition size and stock of pollutant over time.

The second case is for lengths of commitment $h \in [182.6, 194.8]$. On this interval, simulations suggest two main results. The stock of pollutant rises during the first three periods of the game, and afterward decreases and converges asymptotically to its steady state. The coalition size is decreasing over time and reaches its steady state after a finite number of periods. It always begins with 7 signatories at the initial period and then falls, remaining at 4 signatories for any subsequent period. The type of dynamics obtained in this case is analogous to that for the cooperative equilibrium because the stock of pollutant approaches its steady state non monotonically. This is illustrated in Figure 2.1 for the case where the length of the period of commitment is equal to $h = 183$. The stock rises from $z_0 = 900$ to $z_2 = 3.2307 \times 10^3$ after 3 periods of commitment. After that, it decreases following the dynamic relation $z_{k+1} = 0.4005 z_k + 1918.4$ and converges to its steady state $\bar{z} = 3200$ asymptotically.

In the case where h is set to infinity at the initial date, we obtain a coalition of seven signatories as the outcome of the membership game.

Another striking outcome of the simulations is that the trajectory of the stock of pollutant for $h = 1$ always lies above that results from any $h < 1$. At every instant, the stock of pollutant resulting from the equilibrium with $h = \infty$ is always greater than the one we could get from the adoption of some finite lengths of commitment.

Since membership is always decreasing over time, we obtain that for a given length of commitment, the largest coalition is generated at the earlier period of the game. We obtain a non-positive relation between the initial coalition size and the length of commitment as illustrated by the bottom graph in Figure 2.2. The reason is that given the initial stock of pollutant z_0, our simulations have shown that $\bar{n}_0(z_0,h)$ is decreasing in h. But, $n^*(z_0,h)$ is the least integer greater than or equal to $\bar{n}_0(z_0,h)$, thus its number integer ceiling $n^*(z_0,h)$ is non-increasing in h.

2.5.2 The length of commitment and the gains from cooperation

We now consider the effect of the length of commitment on the gain from cooperation.[4] It is useful to distinguish different concepts of gain. The potential gain from cooperation (POC) is defined as the difference between the sum of the discounted net benefits from cooperative and non-cooperative equilibrium. It is given by :

$$POC = V_i(N, z_0, h) - V_j(0, z_0, h).$$

The other concept of gain compares the partial cooperative equilibrium to the non-cooperative equilibrium. Assume a coalition of $n^*(z_0,h)$ signatories. As seen in Section 3.2.1, this results in a decrease in the emission level of signatories and in the aggregate

4. The simulations are done at $t = 0$ over h for the initial period only. But since the rates of emission at each period depend only on the stock of pollution and not explicitly on calendar time, the qualitative results are the same for each subsequent period.

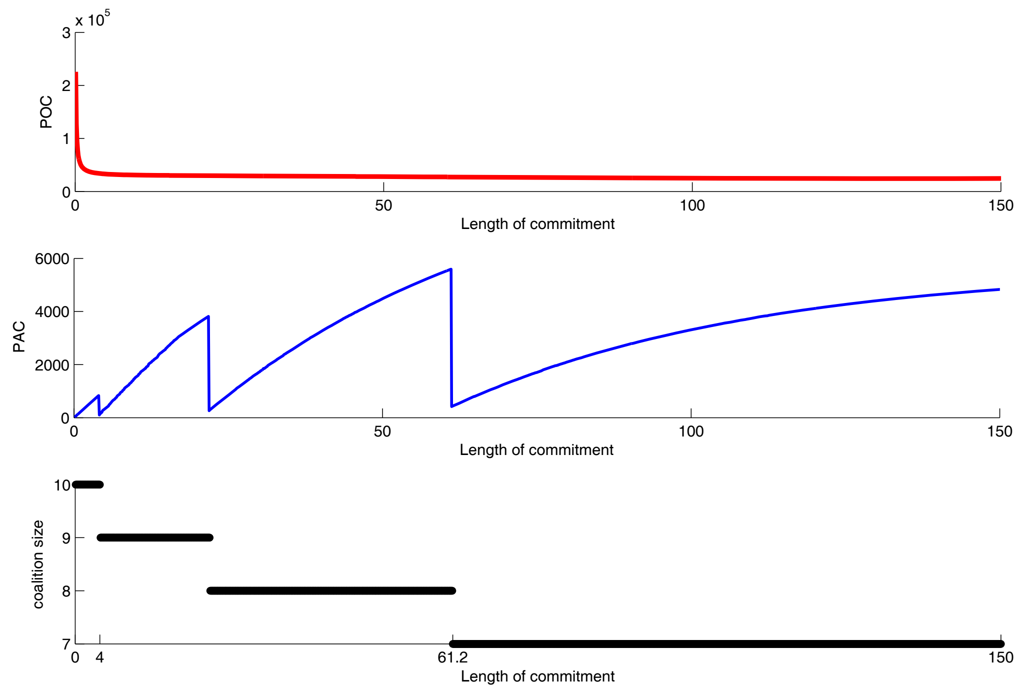

Figure 2.2 – *POC*, *PAC* and coalition size as functions of *h*.

emissions by all countries, as compared to the non-cooperative equilibrium. But both signatories as well as non-signatories gain from the reduction of the global emissions. The partial gain from cooperation by signatories (PAC_s), defined as the difference between the sum of the discounted net benefits by signatories in a stable IEA and in the non-cooperative equilibrium, is given by :

$$PAC_s = V_i(n^*(z_0,h),z_0,h) - V_j(0,z_0,h).$$

The partial gain from cooperation by non-signatories (PAC_{ns}) is defined as the difference between the sum of the discounted net benefits by non-signatories given a stable IEA and that in the non-cooperative equilibrium. It is given by :

$$PAC_{ns} = V_j(n^*(z_0,h),z_0,h) - V_j(0,z_0,h).$$

The average partial gain from cooperation by all countries (PAC) is defined as the mean of the PAC_s and the PAC_{ns} with the respective weights $n^*(z_0,h)/N$ and $1-n^*(z_0,h)/N$. Formally, it is given by :

$$PAC = \frac{n^*(z_0,h)}{N}PAC_s + (1-\frac{n^*(z_0,h)}{N})PAC_{ns}.$$

By the definition of external stability, if $n^*(z_0,h) = 0$ no country can be made better off by cooperating. Hence, in that case, we set $PAC_s = PAC_{ns} = PAC = 0$.

In Figure 2.2, the top graph illustrates that POC is a decreasing function of the length of commitment and that it has a limit which is the outcome with $h = \infty$.

The bottom graph shows the variations of the initial coalition size with respect to the length of commitment. The model generates a maximum level of cooperation of 10 countries at the initial period for $h \in (0,4]$, while a non-positive relation holds for $h \in (4,61.2)$ between the initial coalition size and the length of commitment. It also illustrates that the coalition size in the initial period remains at seven signatories for all $h \geq 61.2$. In particular, taking the limit of $n^*(z_0,h)$ as h goes to infinity, we obtain a seven-signatories coalition as the outcome of the $h = \infty$ coalition size. It is interesting to note that while in the static model of Barrett (1994a) an IEA may result in a significant level of cooperation only if the *POC* is very small, in our model this pessimistic result need not hold for some values of the length of commitment. Indeed, as shown in Figure 2.2, the length of commitment that maximizes the *POC* also sustains the highest level of cooperation.

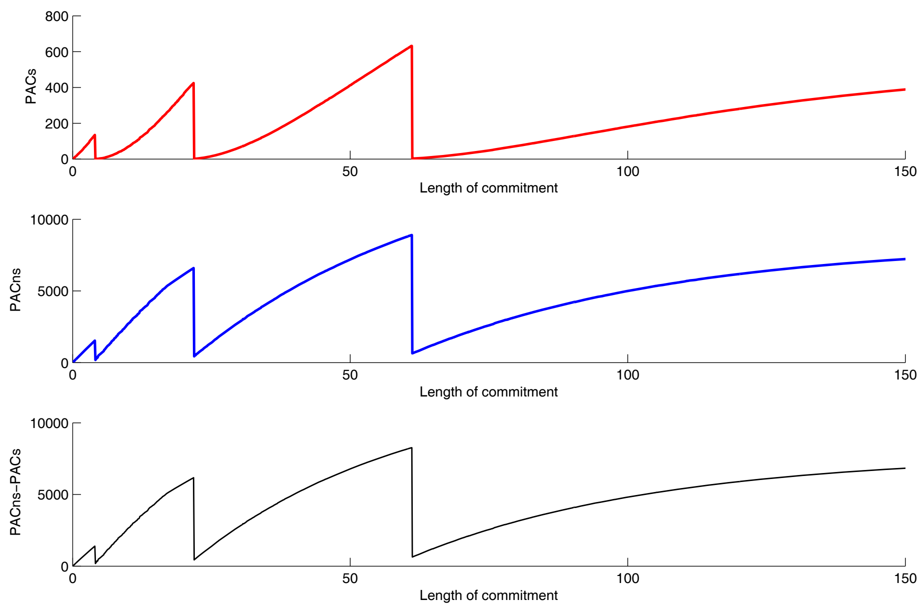

Figure 2.3 – Gain from cooperation by signatories and by non-signatories as functions of h.

The PAC_s, PAC_{ns} and the *PAC* are all piecewise increasing function of h as illustrated in Figure 2.2 and Figure 2.3. They depend on the coalition size, and each of them has as limit the outcome with $h = \infty$. Because non-signatories gain more than signatories from any cooperation, it follows that $PAC_s \geq PAC_{ns}$ for all values of the length of commitment as shown in the bottom graph of Figure 2.3.

In spite of the fact that we cannot claim any general result, the above suggests that for some lengths of commitment $h \neq 1$ the gain from cooperation is higher than for $h = 1$, and that some finite lengths of commitment can, for each of *POC*, *PAC*, PAC_s and PAC_{ns}, sustain a higher value than by letting the length of commitment go to infinity. It is clear that the length of commitment significantly affects the size of stable coalitions and the gains from cooperation.

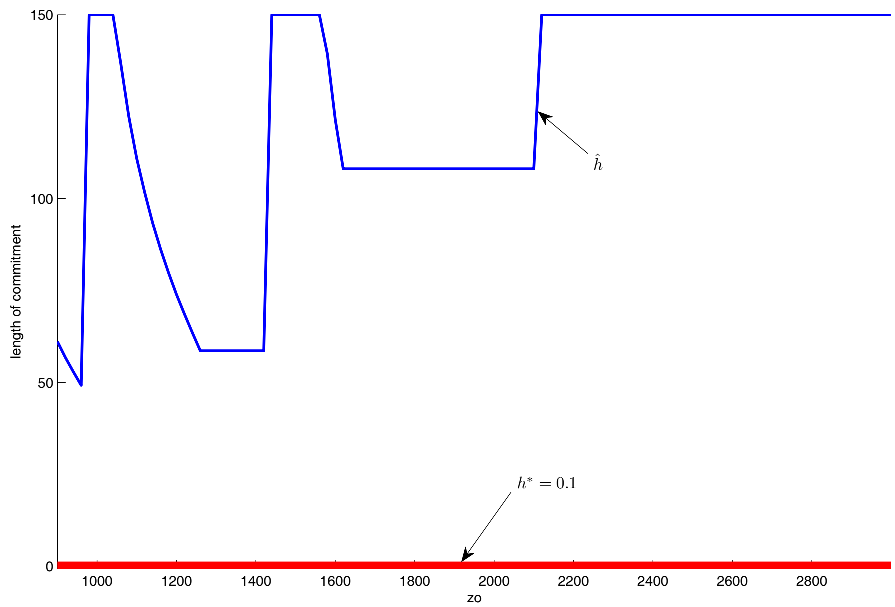

Figure 2.4 – $\hat{h} = \arg max_h PAC_s$ versus $h^* = \arg min_h Q$.

Finally, it is useful to examine the relation between the initial stock of any given period of commitment (z_k), the length of commitment which maximizes the PAC_s (denote it $\hat{h}$) and the length of commitment which can sustain the minimum aggregate emissions (denote it h^*). To do this, we have first simulated 8001 values of the current stock of pollutant following the replication $z_k = z_{k-1} + 20, k = 1, ..., 105$; $z_0 = 900$. For each of those values, we calculate h^* and $\hat{h}$. The top curve of Figure 2.4 illustrates the fact that $\hat{h}$ is a piecewise continuous function of the initial stock of the period of commitment. This is because its argument $n^*(z_0, h)$ is a piecewise function. The bottom curve shows that h^* is independent of the initial stock of pollutant of the period of commitment. Furthermore, $\hat{h}$ is always greater than h^*. These results highlight the difficulties of reconciling the private gain from cooperation and the best protection of the environment.

2.6 Conclusion

The existing literature on dynamic International Environmental Agreements has relied on one of two approaches. The first consists in assuming that membership and emission strategies are determined once and for all, as a function of time, at the outset of an infinite horizon. The other consists in analyzing the problem in a discrete-time framework and assuming that membership and emission decisions are revised at the beginning of each period, whose length has been arbitrarily set equal to one. This paper has explored the middle ground by treating the length of the period of commitment as a positive parameter and studying the effect of varying this parameter on the size of stable International Environmental Agreements. It has been shown that the length of the period of commitment can have considerable impact on the size of stable International Environmental Agreements. The results suggest that for very large lengths of commitment, only small stable coalitions can be sustained. But, below some threshold, as the length of the period of commitment is decreased, the size of the stable coalition tends to increase. It does so until, if this length is sufficiently small, the largest level of cooperation can be attained.

Since our results rest on particular functional forms and on numerical simulations, there is no claim to generality. But they do show clearly that the length of the period of commitment can have very significant effects on the outcome of International Environmental Agreements. This suggests that considerable attention should be devoted to the determination of the length of the period of commitment in discussions of this type of international treaties.

For the purpose of this paper, it has been sufficient to treat the length of commitment as a parameter. However, how best to determine the length of commitment is another matter, which is clearly worthy of further research.

CHAPITRE 3

TRADE STRUCTURE, TRANSBOUNDARY POLLUTION AND MULTILATERAL TRADE LIBERALIZATION : THE EFFECTS ON ENVIRONMENTAL TAXES AND WELFARE

Abstract

This paper considers a trade situation where the production activities of potentially heterogeneous countries generate pollution which can cross borders and harm the well-being of all the countries involved. In each of those countries the policy maker levies pollution taxes on the polluting firms and a tariff on imports in order to correct that distortion. The purpose of the paper is to investigate the effect of a reduction in the tariff on equilibrium pollution taxes and welfare. The existing literature has investigated this problem for trade between two identical countries. This paper analyzes the problem in the more realistic context where countries are not necessarily identical and trade can be multilateral. It becomes possible to show what bias is introduced when those two realities are neglected. I find that a tariff reduction can actually lower output ; it can also lower welfare even if pollution is purely local.

3.1 Introduction

There is a growing concern among environmentalists about the negative effects of freer international trade on environment. The central point is that competitive pressures incurred by freer trade may oblige governments to dilute their environmental instrument. What is unfortunate is that despite the large difference among countries, papers that investigate the impact of trade liberalization on pollution taxes and welfare work only under the restrictive assumption of identical countries. But, we frequently observe that "small" countries trade with "big" partners. In such situations, taking into account the trade structure is important to best characterize the equilibrium.

The goal of this paper is to introduce an asymmetry into a model of trade in open economies, and investigate the effects of this asymmetry on the outcomes of a multilateral-trade liberalization. More accurately, we consider a finite number of trading countries divided into two groups. Countries are identical within group but differ between groups by the number of firms in their industry. The number of firms in each group can differ. We will assume that in each country production entails pollution and that a fraction of pollution emitted in the country flows into the other countries. The governments use tariffs on imports and pollution taxes in order to correct the distortion created by this global pollution. We are interest in how, in this context a tariff reduction can affect the equilibrium output, the equilibrium pollution taxes and the equilibrium social welfare.

The problem described above will be modeled as an oligopolistic trade game where the tariff will be assumed to be the same for all the countries. In a first stage, in each country, the relevant authority chooses unilaterally the pollution tax that maximizes the social welfare of the country. In a second stage, given the tariff on export and the pollution tax rates, each firm decides how much to produce for the home market and how much for the foreign market.

A number of studies have examined the issue of global pollution in an international oligopolistic setting. Among them, Barrett (1994b), Kennedy (1994) and Markusen (1975) ask how strategic environmental policies compared to the first best outcome. Their common result is that the pollution taxes set unilaterally are not in general socially optimal. Those studies assume free trade and identical countries in their analysis. In this paper, we relax these two assumptions and focus the analysis on the effects of multilateral-trade liberalization on the equilibrium pollution taxes, the equilibrium output and the equilibrium welfare of the countries.

The model used is closely related to that of Burguet and Sempere(2003) and to that of Baksi and Chaudhuri (2009). Burguet and Sempere(2003) explore the impacts of a uniform tariff reduction on welfare and environmental policy. They show that a bilateral tariff reduction can affect environmental policy through two channels. First, they find that a bilateral tariff reduction always increases output which in turn lowers price and increases marginal damages of output. This incites governments to raise their environmental protection level by increasing the pollution taxes. Second, a bilateral tariff reduction diminishes revenues from imports and reduces the cost of exports, hence encouraging governments to dilute their environmental protection. The net effect of a tariff reduction on environmental policy depends on which channel outweighs the other. They also show that when the environment policy is a pollution tax, a bilateral tariff reduction always improves welfare. The limitations of that paper are that it considers only bilateral trade between identical countries, it assumes a monopoly in each country, and considers local pollution only.

Baksi and Chaudhuri (2009) extend that paper to an arbitrary number of firms in each of the two trading countries and also allow for many types of pollution. On the one hand, they show that trade liberalization always increases the output level in each country. It also increases the pollution tax when pollution is sufficiently harmful. On the other hand, they find that the trade liberalization always improves social welfare when pollution is purely local.

In this paper, as in Baksi and Chaudhuri (2009), we consider varying degree of spillover of pollution to other countries, going from purely local pollution to totally global pollution. But our approach is more general in some key respects : there is an arbitrary number of countries involved in trade ; there is an arbitrary number of countries divided into two groups according to the number of firms in their industry ; the number of countries in each group can differ. We focus on the impacts of this type of asymmetry on multilateral environmental policies.

To do this, we derive the Nash equilibrium pollution taxes, the equilibrium output and the equilibrium social welfare. We examine the effects of a tariff reduction on these equilibrium outcomes and compare them with those obtained when all the countries are identical. More accurately, we compare the results from the situation in which the two types of trading partners coexist on the world market to the one where all the trading partners are identical.

Unlike the case of identical countries in which trade liberalization always increases output, two situations may arise when the two types of countries coexist. Trade liberalization may increase output of the countries in one group while lowering output of the countries in the other group. It may also increase the output of all the countries. As in Baksi and Chaudhuri (2009), we also find that in the identical countries setting, trade liberalization increases the pollution taxes when the pollution is sufficiently harmful. Moreover, social welfare is concave in the trade tariff and trade liberalization always increases social welfare when the pollution is purely local. However, in the presence of asymmetry, these results may not hold, depending on the range of asymmetry and the number of actors involved in trade.

The remainder of this paper is organized as follows. Section 3.2 sets out and solves the model. Section 3.3 presents the outcomes of the model obtained with identical countries. Section 3.4 compares the results of the asymmetric model to those derived with identical countries. Section 3.5 concludes.

3.2 The model

Consider a world of $N \geq 1$ countries, divided in two groups. Countries are identical within group but differ between group by the number of firms in their industry. The first group is made of N_1 countries and the industry of each country in that group has n_1 firms. The second group is constituted of N_2 countries and each country in that group has n_2 firms. Industries are assumed to produce a single homogenous good. They use the same technology of production and c is their constant marginal cost of production.

A single firm that resides in a country j produces and ships y_j^i quantity of good on the market of the country i. For simplicity, there is not storage. Firms compete in quantities in the market of their own country and in each foreign market, like in the reciprocal dumping game by Brander and Krugman (1983). The inverse demand is the same for countries in both groups and is given by

$$P(y^i) = a - y^i; \;\; a > c, \tag{3.1}$$

where y^i denotes the total quantity demanded in country i.

Each country levies a tariff z on each unit of import from foreign countries. The tariff is exogenous and is the same in both group of countries. Multilateral trade liberalization is defined as a uniform reduction of the tariff in the N countries. [1]

During their activity of production, firms emit pollution that damages a shared environmental resource. It is assumed that one unit of production generates one unit of pollution and that pollution is transboundary. We denote by a parameter $\lambda \in [0,1]$ the fraction of pollution emitted in one country that damages the others countries with $\lambda = 0$ being strictly local pollution and $\lambda = 1$ being perfectly global pollution.

The damage cost function from the pollution of country j is assumed to be quadratic, convex, and increasing in the pollution level :

$$D_j = \frac{\gamma}{2}\left(y_j + \lambda \sum_{k \in N_1 \setminus \{j\}} y_k + \lambda \sum_{k \in N_2 \setminus \{j\}} y_k\right)^2,$$

where $\gamma \geq 0$ is the damage cost parameter and, for all $k = 1, \ldots, N$,

$$y_k = n_i \sum_{i_1 \in N_1} y_k^{i_1} + n_i \sum_{i_2 \in N_2} y_k^{i_2}, \tag{3.2}$$

is the total output produced in the country k ; where, $i = 1$ if $k \in N_1$ and $i = 2$ if $k \in N_2$.

The environmental instrument in each country is a pollution tax imposed by its government to its domestic firms. Denote by t_i the pollution tax per unit of pollution in the country i.

In a first stage, the relevant authority in each country decides the tax level that maximizes the country's social welfare considering as given the tax level of the remaining countries. It also considers the common tariff z (per unit of export) in both groups of country as given. In a second stage, each firm decides the output level that maximizes its profits. In that decision, it considers the output level of the remaining $n_1N_1 + n_2N_2 - 1$ firms and the set of taxes in both groups of countries as given. The subgame perfect Nash equilibrium is derived using the backward induction framework.

1. This situation prevails for instance in NAFTA where member countries are asked to diminish uniformly their trade tariff over a defined calendar of time.

3.2.1 The second stage of the game : output decision of firms

The typical firm that operates in country j chooses the output strategy $\{y_j^i\}_{i=1}^{i=N}$ that maximizes its profit, namely :

$$\max_{\{y_j^i\}_{i=1}^{i=N}} \sum_{i=1}^{N} y_j^i(a-y^i) - \sum_{i=1,i\neq j}^{N} zy_j^i - (c+t_j)\sum_{i=1}^{N} y_j^i, \tag{3.3}$$

where

$$y^i = n_1 \sum_{j_1 \in N_1} y_{j_1}^i + n_2 \sum_{j_2 \in N_2} y_{j_2}^i \tag{3.4}$$

denotes the total quantity sold in country i, for $i = 1,...,N$.

Assuming an interior solution, the first-order conditions for this problem are :

$$a - y^i - y_j^i = z + c + t_j, \quad \forall i \neq j \tag{3.5}$$

$$a - y^j - y_j^j = c + t_j, \quad \forall j = 1,...,N. \tag{3.6}$$

Define the home (export) augmented marginal cost as the marginal cost plus tax t_j (marginal cost plus tax t_j and tariff z). The LHS of (3.5) and (3.6) are respectively marginal revenue from export and from home production. Those fist-order conditions above say that the given firm allocates to export (home) the output level for which the marginal revenue of production for export (home) equals to its export (home) augmented marginal cost of production.

Solving the system of equations (3.4), (3.5) and (3.6), we obtain the total sales of the good for country i_1 in the first group :

$$y^{i_1} = [(n_1N_1 + n_2N_2)(a-c) - z(n_1(N_1-1) + n_2N_2) - n_1 \sum_{j_1 \in N_1} t_{j_1} - n_2 \sum_{j_2 \in N_2} t_{j_2}]/d, \tag{3.7}$$

where $d = 1 + n_1N_1 + n_2N_2$. Using a similar reasoning, we verify that total sales of the good for country i_2 in the second group is :

$$y^{i_2} = [(n_1N_1 + n_2N_2)(a-c) - z(n_1N_1 + n_2(N_2-1)) - n_1 \sum_{j_1 \in N_1} t_{j_1} - n_2 \sum_{j_2 \in N_2} t_{j_2}]/d. \tag{3.8}$$

Substituting (3.7) into (3.6), we get the quantity produced and consumed in a country i of the first group :

$$y_i^i = -t_i + \{a - c + z[(N_1 - 1)n_1 + n_2 N_2] + n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}\}/(1 + n_1 N_1 + n_2 N_2). \tag{3.9}$$

Similarly, substituting (3.8) into (3.6), we obtain the quantity of good produced and consumed in a country i of the second group :

$$y_i^i = -t_i + \{a - c + z[n_1 N_1 + (N_2 - 1)n_2] + n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}\}/(1 + n_1 N_1 + n_2 N_2). \tag{3.10}$$

Now, substituting (3.7) into (3.5), we derive the quantity of the good produced by a firm in country j and shipped to country i of the first group :

$$y_j^i = -t_j + \{a - c - (1 + n_1)z + n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}\}/(1 + n_1 N_1 + n_2 N_2), \text{ for all } j \neq p. \tag{3.11}$$

Substituting (3.8) into (3.5) yields the quantity of good produced by a firm in a given country j and shipped to a given country i belonging to the second group :

$$y_j^i = -t_j + \{a - c - (1 + n_2)z + n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}\}/(1 + n_1 N_1 + n_2 N_2), \text{ for all } j \neq p. \tag{3.12}$$

Substituting (3.9)-(3.12) in (3.2), we get the total output produced by a country j_1 in the first group :

$$y_{j_1} = -n_1 N t_{j_1} + n_1 N(a - c)/d - (N - 1)n_1 z/d + n_1 N[n_1 \sum_{j \in N_1} t_j + n_2 \sum_{j \in N_2} t_j]/d. \tag{3.13}$$

Likewise, plugging (3.9)-(3.12) in (3.2) we obtain the total output produced by a country j_2 in the second group which is given by :

$$y_{j_2} = -n_2 N t_{j_2} + n_2 N(a - c)/d - (N - 1)n_2 z/d + n_2 N[n_1 \sum_{j \in N_1} t_j + n_2 \sum_{j \in N_2} t_j]/d. \tag{3.14}$$

From (3.13) and (3.14) we observe that while an exogenous increase of the national tax always lowers the national production, an exogenous increase of the foreign taxes raises the national production.

Each country's net import is the difference between its total consumption and its total production. Using (3.7), (3.8), (3.13) and (3.14), we derive the expressions for the net import of each country in the first and in the second group, which are respectively given by :

$$y^{i_1} - y_{i_1} = n_1 N t_{i_1} - (1+n_1 N)[n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}]/d + (a-c-z)N_2(n_2 - n_1)/d,$$

$$y^{i_2} - y_{i_2} = n_2 N t_{i_2} - (1+n_2 N)[n_1 \sum_{j_1 \in N_1} t_{j_1} + n_2 \sum_{j_2 \in N_2} t_{j_2}]/d + (a-c-z)N_1(n_1 - n_2)/d.$$

In each country, the net import is increasing in its own pollution tax and is decreasing in the foreign pollution tax. Note that in the case of symmetric industry size ($n_1 = n_2$), the net import does not depend on the tariff. However, in the case of asymmetric industry sizes ($n_1 \neq n_2$), the net imports of countries with higher industry size are affected negatively by a tariff reduction, while the reverse is true for countries with the lower industry size. This is the extension to asymmetry of a result by Baksi and Chaudhuri (2009) and Burguet and Sempere(2003). Recall that both papers investigate the effects of a tariff reduction on the optimal pollution tax for two identical trading countries. They find, among other things that the net import of each country does not depend on the trade tariff.

3.2.2 First stage : environmental policy

In the first stage of the game, the government of each country chooses the pollution tax that maximizes the country's welfare, considering as given the tariff level and the pollution tax of the other countries. [2] Welfare for each country is the sum of the consumer surplus, the producer surplus, the tariff revenue and the pollution tax revenue, minus the

2. This results in a Nash equilibrium pollution tax which is not in general socially efficient as pointed out by Kennedy (1994).

pollution damage. Its expression for the country $j \in N_1 \cup N_2$ is given by : [3]

$$SW_j(t_j, t_{-j}) = CS_j + PS_j + TR_j + ER_j - D_j, \tag{3.15}$$

where
$PS_j = n_k \sum_{i_1 \in N_1} (y_j^{i_1})^2 + n_k \sum_{i_2 \in N_2} (y_j^{i_2})^2$ is the producer surplus,
$TR_j = zn_1 \sum_{k_1 \in N_1 \setminus \{j\}} y_{k_1}^j + zn_2 \sum_{k_2 \in N_2 \setminus \{j\}} y_{k_2}^j$ is the tariff revenue,
$ER_j = t_j n_k \sum_{i_1 \in N_1} y_j^{i_1} + n_k t_j \sum_{i_2 \in N_2} y_j^{i_2} = t_j y_j$ is the pollution tax revenue,
$CS_j = \frac{1}{2}(n_1 \sum_{k_1 \in N_1} y_{k_1}^j + n_2 \sum_{k_2 \in N_2} y_{k_2}^j)^2 = \frac{1}{2}(y^j)^2$ is the consumer surplus, where the y_j^i, y^j are given by (3.7)-(3.12), and where $k = 1$ if $j \in N_1$ and $k = 2$ if $j \in N_2$.

The first-order conditions for (3.15) yield the best-response pollution tax for country j. The expression for the equilibrium tax of country j, $t_j(t_{-j})$, depends on the taxes of the other countries and on the parameters of the model. The second-order condition for the welfare maximization is verified since we have the following inequality :

$$\frac{\partial^2 SW_j}{\partial (t_j)^2}(t_j, t_{-j}) = A_k - \gamma n_k^2 N^2 [-1 + \frac{n_k(1-\lambda) + \lambda(n_1 N_1 + n_2 N_2)}{1 + n_1 N_1 + n_2 N_2}]^2 < 0, \tag{3.16}$$

where $A_k = n_k^2[1 + 2(N_1 + N_2)(n_k - 1 - n_1 N_1 - n_2 N_2)]/(1 + n_1 N_1 + n_2 N_2)^2 < 0$, and where $k = 1$ if $j \in N_1$ and $k = 2$ if $j \in N_2$. [4]

The tax policies at the equilibrium for the first group and the second group of countries are given respectively by : [5]

$$t_1 = [z(v_1 \hat{e}_2 - e_2 \hat{v}_1) + (a-c)(v_2 \hat{e}_2 - e_2 \hat{v}_2)]/(e_1 \hat{e}_2 - e_2 \hat{e}_1), \tag{3.17}$$

$$t_2 = [z(\hat{v}_1 e_1 - \hat{e}_1 v_1) + (a-c)(\hat{v}_2 e_1 - \hat{e}_1 v_2)]/(e_1 \hat{e}_2 - e_2 \hat{e}_1), \tag{3.18}$$

where, $e_i, \hat{e}_i, v_i, \hat{v}_i$ for all $i = 1, 2$ are given in Appendix.

In each country, the equilibrium pollution tax results in the strategic interaction of

3. In the expression $SW_j(t_j, t_{-j})$, t_{-j} represents the vector of taxes of the countries other than j.
4. $A_1 < 0$ indeed : in its expression, denote by $g(n_1)$ the quantity in square brackets. Since $g'(n_1) = 2(N_1 + N_2)(1 - N_1) \leq 0$ and $g(1) = 1 - 2(N_1 + N_2)(N_1 + n_2 N_2) < 0$, it follows that $g(n_1) < 0$ for all $n_1 \geq 1$. Using a similar reasoning, we can show that $A_2 < 0$.
5. We show in the Appendix how to derive the expressions of equilibrium taxes (3.17) and (3.18).

three sources of market failure. First, the rent capture effect that tends to lower the equilibrium pollution tax from its globally efficient level. Since the market is imperfect, each government has the incentive to provide an edge to its domestic firms so that they can gain more rent through their exports. Second, the pollution-shifting effect increases the equilibrium pollution taxes as each country tends to shift output and its associated pollution to the foreign countries. Third, the transboundary externality effect that tends to lower the equilibrium pollution tax, as each country does not care about the damages associated to its pollution on the well being of the other countries. For an overview of these effects, see for instance Kennedy (1994).

3.3 Symmetric equilibrium

Setting in (3.17) $N_1 = N$, $N_2 = 0$ and $n_1 = n$, we get the tax level for the symmetric equilibrium, which is given by :

$$t_s = \frac{(N-1)\{1+nN(1+n)-\gamma nN[1+\lambda(N-1)][1+n(N-1)(1-\lambda)]\}z+\tau_s}{nN^2[1+n(N-1)+\gamma(1+\lambda(N-1))+n(N-1)\gamma(1-\lambda^2)]}, \quad (3.19)$$

where the subscript s stands for the symmetric equilibrium and where
$\tau_s = (a-c)N\{n-1-nN+\gamma nN[1+\lambda(N-1)][1+n(N-1)(1-\lambda)]\}$.

Substituting (3.19) into either (3.7) or (3.8) , we get the total production of each country when all the countries are identical, given by :

$$y_s = \frac{(a-c)(1+n(N-1))-(N-1)(1+nN)z}{N[1+\gamma(1-\lambda+N\lambda)+n(N-1)(1+\gamma(1-\lambda)(1+\lambda(N-1)))]}.$$

Since y_s is linear in z and has a negative slope, a reduction of the tariff results in an increase of the national production of each country.

3.3.1 Effect of tariff reduction on the equilibrium tax : the symmetric case

Using (3.19), we derive

$$\frac{\partial t_s}{\partial z} = \frac{(N-1)\{1+nN(1+n)-\gamma nN[1+\lambda(N-1)][1+n(N-1)(1-\lambda)]\}}{nN^2[1+n(N-1)+\gamma(1+\lambda(N-1))+n(N-1)\gamma(1-\lambda^2)]}. \quad (3.20)$$

Since the denominator of (3.20) is positive, the sign of that expression is the same as that of its numerator. Solving the equation $\frac{\partial t_s}{\partial z} = 0$ for the rate of transboundary pollution, λ, we obtain the following roots :

$$\underline{\lambda} = [\gamma n N(N-1)(1+n(N-2)) - \sqrt{\Delta}]/(2n^2\gamma N(N-1)^2), \tag{3.21}$$

$$\overline{\lambda} = [\gamma n N(N-1)(1+n(N-2)) + \sqrt{\Delta}]/(2n^2\gamma N(N-1)^2), \tag{3.22}$$

where
$\Delta = N\gamma(n(N-1))^2(N\gamma(1+nN)^2 - 4(1+nN(1+n)))$.
The above roots are real if and only if $\Delta \geq 0$. This last condition is equivalent to

$$\gamma \geq \frac{4(1+nN(1+n))}{N(1+nN)^2} \equiv \gamma_1 \tag{3.23}$$

Furthermore $\underline{\lambda} \geq 0$ if and only if

$$\gamma \leq \frac{1+nN(1+n)}{nN(1+n(N-1))} \equiv \gamma_2 \tag{3.24}$$

and $\overline{\lambda} \leq 1$ if and only if

$$\gamma \leq \frac{1+nN(1+n)}{nN^2} \equiv \gamma_3 \tag{3.25}$$

These computations lead to the following proposition.

Proposition 11. *Under symmetry, we have :* (i) *if* $\gamma < \gamma_1$ *then* $\frac{\partial t_s}{\partial z} > 0$. *When the damage cost parameter is sufficiently small, multilateral trade liberalization lowers the equilibrium pollution tax, regardless to the remaining feasible parameters of the model.* (ii) *If* $\gamma \in [\gamma_1, \gamma_2]$, *then* $\frac{\partial t_s}{\partial z} > 0$ *if and only if* $\lambda \leq \underline{\lambda}$ *or* $\lambda \geq \overline{\lambda}$. (iii) *if* $\gamma \in [\gamma_2, \gamma_3]$, *then* $\frac{\partial t_s}{\partial z} > 0$ *if and only if* $\lambda \geq \overline{\lambda}$. (iv) *if* $\gamma > \gamma_3$ *then* $\frac{\partial t_s}{\partial z} < 0$.

Notice that the above thresholds of the damage cost parameter have the following features. First they satisfy the inequalities $\gamma_3 \geq \gamma_2 \geq \gamma_1$. They are also decreasing functions of the number of countries N participating in trade. In addition, each of them goes to zero as N goes to infinity. Since only case (iv) of Proposition 11 is likely to hold when each γ_i goes to zero, the result follows. For the symmetric equilibrium, when the number of countries involved in trade becomes sufficiently large, multilateral trade liberalization is more likely to increase the environmental pollution tax. The result (iv) from Proposition 11 can be seen as the "mitigation effect". Indeed, it states that if the damages are too harmful, countries must raise their environmental tax in response to a tariff reduction. This in turn will lower the national production of the dirty good in each country (see Eq 3.13 or Eq 3.14). The overall effect will be the mitigation of the damages incurred from the global pollution.

3.3.2 Effect of tariff reduction on welfare : the symmetric case

Substituting (3.19) in (3.15), we can derive SW_s, the expression for the welfare of a country in the symmetric setting. That expression depends on the tariff z and the remaining parameters of the model. Its derivative with respect to z is given by

$$\frac{\partial SW_s}{\partial z} = \frac{(N-1)^2(1+nN)^2[(a-c)N\gamma\lambda(1-\lambda+N\lambda)-z(1+\gamma(1+\lambda(N-1))^2)]}{N^2[-1+\gamma(-1+\lambda-N\lambda)-n(N-1)(1+\gamma(1-\lambda)(1+\lambda(N-1)))]^2}. \tag{3.26}$$

Proposition 12. *Under symmetry, there exist a tariff threshold* $\hat{z} \equiv \frac{(a-c)N\gamma\lambda[1+\lambda(N-1)]}{1+\gamma[1+\lambda(N-1)]^2}$, *under which a reduction of the tariff lowers the well-being of each country. Above that threshold, a reduction of the tariff improves the well-being.* $(i.e. \frac{\partial SW_s}{\partial z} \leq 0$ *if and only if* $z \geq \hat{z}$*).*

In the case of the purely local pollution $\hat{z} = 0$, which implies that a reduction of the tariff always increases the payoff of each country. Furthermore, we have :

$$\frac{\partial \hat{z}}{\partial N} = (a-c)N\gamma\lambda \frac{1+(2N-1)\lambda+\gamma(1-\lambda)(1+\lambda(N-1))^2}{[1+\gamma(1+\lambda(N-1))^2]^2} > 0 \text{ for } \lambda\gamma > 0.$$

This inequality, combined with the results of Proposition 12, implies that for a spillover pollution problem, as the number of countries involved in trade increases, trade liberalization is less likely to improve the well being of each country.

This section yields exactly the same results as in Baksi and Chaudhuri (2009) for the particular case where $N = 2$. It is then a pure extension of the paper of both authors to the case where an arbitrary number of countries are involved in trade. We next investigate the role of asymmetry.

3.4 Effects of asymmetry

This section allows for the number of countries and firms to differ across groups. We are interested in the effects of having the two groups of countries involved in trade, rather than having all the firms and the countries identical. To do this, we derive the equilibrium under asymmetry and we compare it with the equilibrium under the symmetric setting calculated in Section 3.3.

3.4.1 Asymmetry and trade liberalization : the effects on output

Substituting (3.17) and (3.18) in (3.13), we obtain the equilibrium output y_{j_1} for each country in the first group, which upon differentiation with respect to z yields :

$$\begin{aligned}\frac{\partial y_{j_1}}{\partial z} =& n_1 N[(1+n_2N_2)(e_2\hat{v}_1 - v_1\hat{e}_2) + n_2N_2(\hat{v}_1 e_1 - \hat{e}_1 v_1)]/[d(e_1\hat{e}_2 - e_2\hat{e}_1)] \\ &+ n_1(N-1)(e_2\hat{e}_1 - e_1\hat{e}_2)/[d(e_1\hat{e}_2 - e_2\hat{e}_1)]. \end{aligned} \tag{3.27}$$

Similarly, substitution (3.17) and (3.18) in (3.14), we get the equilibrium output y_{j_2} for each country in the second group, which upon differentiation with respect to z gives :

$$\begin{aligned}\frac{\partial y_{j_2}}{\partial z} =& n_2 N[(1+n_1N_1)(\hat{e}_1 v_1 - \hat{v}_1 e_1) + n_1N_1(v_1\hat{e}_2 - e_2\hat{v}_1)]/[d(e_1\hat{e}_2 - e_2\hat{e}_1)] \\ &+ n_2(N-1)(e_2\hat{e}_1 - e_1\hat{e}_2)/[d(e_1\hat{e}_2 - e_2\hat{e}_1)]. \end{aligned} \tag{3.28}$$

Trade liberalization increases output for each country in the first group if and only (3.27) is negative. It must do so for each country in the second group if and only if (3.28) is negative. Notice that (3.27) and (3.28) can have the same or opposite signs depending on the distribution of firms across groups, the size of the groups, the damage cost parameter and the degree of spillover of the pollution. In particular they can take positive values. So, contrary to the case where countries are identical, trade liberalization can actually lower or increases output for all the countries. It may increase output of the countries in one of the two groups and lower it for the countries in the other group.

To better understand the implications of asymmetry, consider the simple case for which $n_1 = 1, N_1 = 2, n_2 = 4$, $N_2 = 6$, for arbitrary values of parameters λ, γ, c and a. For this case, the marginal output for each country of type 1 and 2 are respectively given by

$$\begin{aligned}\frac{\partial y_{j_1}}{\partial z} =& 7[-1719+4\gamma(12023-28841\lambda+18121\lambda^2) \\ & +256\gamma^2(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)]/108\rho(\lambda,\gamma) \qquad (3.29)\\ \frac{\partial y_{j_2}}{\partial z} =& 7[-1371-4\gamma(102706-196501\lambda+94507\lambda^2) \\ & +1024\gamma^2(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)]/108\rho(\lambda,\gamma) \qquad (3.30)\end{aligned}$$

where,

$\rho(\lambda,\gamma) = 95 - \gamma(-19231 + 36938\lambda - 17739\lambda^2) - 32\gamma^2(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)$.

Notice that $\rho(\lambda,\gamma)$ is positive because it is the sum of three positive terms. Likewise, each term constituting the numerator of $\frac{\partial y_{j_2}}{\partial z}$ is negative so that $\frac{\partial y_{j_2}}{\partial z} < 0$. Thus, multilateral trade liberalization always increases the equilibrium output of each country in the second group.

Since $\rho(\lambda,\gamma) > 0$, the sign of $\frac{\partial y_{j_1}}{\partial z}$ depends on that of its numerator. The study of that sign suggests three possible cases for the transboundary pollution parameter, λ.

The first case corresponds to perfectly global pollution (*i.e* $\lambda = 1$). In that case, making use of (3.29), we get $\frac{\partial y_{j_1}}{\partial z} > 0$ if and only if $\gamma > 0.33$. Hence, multilateral trade liberalization lowers the equilibrium output of each country in the first group only when the damage cost parameter is large.

The second case is for extreme values of the transboundary pollution parameter (*i.e.* $\lambda \in [0, 0.108) \cup (0.913, 1)$). In this case, the numerator of (3.29) is positive if and only if we have :

$$\gamma_{i_1}(\lambda) < \gamma < \gamma_{i_2}(\lambda),$$

where

$$\gamma_{i_1}(\lambda) = \frac{-2(12023 - 28841\lambda + 18122\lambda^2) + \sqrt{\Delta_y(\lambda)}}{256(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)},$$
$$\gamma_{i_2}(\lambda) = \frac{-2(12023 - 28841\lambda + 18122\lambda^2) - \sqrt{\Delta_y(\lambda)}}{256(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)},$$

and where,

$\Delta_y(\lambda) = 4(12023 - 28841\lambda + 18122\lambda^2)^2 + 1719 \times 256(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)$. Notice that $\gamma_{i_1}(\lambda)$ and $\gamma_{i_2}(\lambda)$ are positive and real. [6] In addition, they verify the inequality $\gamma_{i_1}(\lambda) < \gamma_{i_2}(\lambda)$ for $\lambda \in [0, 0.108) \cup (0.913, 1)$.

From the above findings, we see that the inequality $\frac{\partial y_{j_1}}{\partial z} > 0$ must hold if and only if $\gamma \in (\gamma_{i_1}(\lambda), \gamma_{i_1}(\lambda))$. Thus, trade liberalization lowers the equilibrium output of each country in the first group only when the damage cost parameter lies in the open interval $(\gamma_{i_1}(\lambda), \gamma_{i_1}(\lambda))$.

The third case corresponds to intermediate values of the transboundary pollution parameter (*i.e.* $0.108 \leq \lambda \leq 0.913$). In this case, we have $\Delta_y(\lambda) < 0$ so that the numerator of (3.29) is negative. Hence multilateral trade liberalization always increases the equilibrium output of each country in the first group in this situation.

In the symmetric setting presented in Section 3.3, we have found that trade liberalization always raises output. For this simple case of asymmetry, trade liberalization increases output only for restrictive values of parameters. Namely, it increases output of all the countries if and only if $\lambda = 1$ and $\gamma < 0.33$ or $\lambda \in [0.108, 0.913]$ or $\lambda \in [0, 0.108] \cup (0.913, 1)$ and $\gamma < \gamma_{i_1}(\lambda)$ or $\lambda \in [0, 0.108] \cup (0.913, 1)$ and $\gamma > \gamma_{i_2}(\lambda)$.

6. We have shown numerically that $\Delta_y(\lambda) \geq 0$ if and only if $\lambda \in [0, 0.108] \cup [0.913, 1]$

3.4.2 Asymmetry and trade liberalization : the effects on pollution taxes

Using (3.17) and (3.18), we derive

$$\frac{\partial t_1}{\partial z} = (v_1\hat{e}_2 - e_2\hat{v}_1)/(e_1\hat{e}_2 - e_2\hat{e}_1), \tag{3.31}$$

$$\frac{\partial t_2}{\partial z} = (\hat{v}_1 e_1 - \hat{e}_1 v_1)/(e_1\hat{e}_2 - e_2\hat{e}_1). \tag{3.32}$$

From expression (3.31), we see that trade liberalization increases the pollution tax of a country in the first group if and only if $(v_1\hat{e}_2 - e_2\hat{v}_1)(e_1\hat{e}_2 - e_2\hat{e}_1) < 0$. Likewise, (3.32) tells us that a tariff reduction lowers the pollution tax of a country in the second group if and only if $(\hat{v}_1 e_1 - \hat{e}_1 v_1)(e_1\hat{e}_2 - e_2\hat{e}_1) > 0$.

Since $v_1, e_1, \hat{e}_1, \hat{e}_2, e_2$ and $\hat{v}_1$ depend on n_1, N_1, n_2, N_2 λ, γ, these results merely show that the impact of trade liberalization on the pollution taxes depends on firm characteristics, country characteristics and on the spillover and the damage cost parameters. So, the distribution of firms across countries as well as the number of countries in each group impact significantly the pollution taxes outcome of trade liberalization.

To better understand this result, consider again the particular case of asymmetry where $n_1 = 1, N_1 = 2, n_2 = 4$ and $N_2 = 6$, for arbitrary parameter values λ, γ, c and a. The expressions (3.31) and(3.32) of marginal taxes become

$$\begin{aligned}\frac{\partial t_1}{\partial z} =& 7[347 + 4\gamma(6770 - 10573\lambda + 3747\lambda^2) \\ & - 384\gamma^2(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)]/96\rho(\lambda,\gamma), \end{aligned} \tag{3.33}$$

$$\begin{aligned}\frac{\partial t_2}{\partial z} =& 7[2329 - \gamma(-526024 + 923324\lambda - 402516\lambda^2) \\ & - 4608\gamma^2(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)]/1152\rho(\lambda,\gamma). \end{aligned} \tag{3.34}$$

Notice that the numerator of $\frac{\partial t_2}{\partial z}$ is always positive as the sum of three positive terms. Hence, multilateral liberalization always diminishes the pollution taxes of the countries in the first group.

The analysis of the signs of $\frac{\partial t_1}{\partial z}$ can be carried out by distinguishing three types of transboundary pollution.

The first case is for totally global pollution (*i.e* $\lambda = 1$). In this case, we get $\frac{\partial t_1}{\partial z} > 0$ if and only if $0 \leq \gamma < 1.549$. Thus a tariff reduction lowers the pollution tax of a country in the first group if and only if the damage cost parameter is smaller than 1.549.

The second case corresponds to extreme values of the transboundary pollution parameters (*i.e* $0 \leq \lambda < 0.175$ or $0.995 \leq \lambda < 1$). In this situation, the numerator of (3.33) is a second degree polynomial in γ. The values of γ that solve the equation $\frac{\partial t_1}{\partial z} = 0$ are given by :

$$\underline{\gamma}(\lambda) = \frac{2(6770 - 10573\lambda + 3747\lambda^2) + \sqrt{\Delta_1(\lambda)}}{384(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)},$$
$$\overline{\gamma}(\lambda) = \frac{2(6770 - 10573\lambda + 3747\lambda^2) - \sqrt{\Delta_1(\lambda)}}{384(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)},$$

where $\Delta_1(\lambda) = 4(6770 - 10573\lambda + 3747\lambda^2)^2 + 133248(-1+\lambda)(1+7\lambda)(-23+22\lambda)(-26+25\lambda)$. We have verified numerically that $\Delta_1(\lambda) \geq 0$ for all $\lambda \in [0, 0.175] \cup [0.995, 1)$. Thus, $\underline{\gamma}(\lambda)$ and $\overline{\gamma}(\lambda)$ are two positive real numbers. [7] So, the numerator of (3.33) is positive if and only if $0 \leq \gamma < \underline{\gamma}(\lambda)$ or $\gamma > \overline{\gamma}(\lambda)$. Recall that the denominators of (3.29),(3.30) and (3.33) are proportional and they are all positive.

Consequently, for ($0 \leq \lambda < 0.175$ or $0.995 \leq \lambda < 1$) multilateral trade liberalization lowers the pollution taxes for a country in the first group if and only if the damage cost parameter is either lower than $\underline{\gamma}(\lambda)$ or it is greater than $\overline{\gamma}(\lambda)$.

The third case corresponds to transboundary pollution parameter in the open interval $(0.175, 0.982)$. In this situation, $\Delta_1(\lambda)$ is always negative so that numerator of (3.33) is positive. Hence, multilateral trade liberalization actually lowers the pollution taxes of the countries in the first group. These results imply that it may be optimal for all the countries to lower their pollution taxes as response to a tariff reduction even if the damage cost parameter is large. Thus the "mitigation effect" identified in Proposition 11 may vanish when asymmetry prevails.

These results can be summarized as follows. Asymmetry entails new strategic consideration on the pollution taxes resulting from a tariff reduction. These effects depend on

7. Indeed, one can easily prove that $\underline{\gamma}(\lambda)$ and $\overline{\gamma}(\lambda)$ have a positive sum as well as a positive product.

the damage cost parameter, the degree of transboundary pollution and the asymmetry of the industry size of countries involved in trade. It can decrease the tax level of all the countries ($\lambda = 1$ and $0 \leq \gamma < 1.549$ or $\gamma > \bar{\gamma}(\lambda)$ and $\lambda \in [0, 0.175] \cup [0.995, 1)$ or $0 \leq \gamma < \underline{\gamma}(\lambda)$ and $\lambda \in [0, 0.175] \cup [0.995, 1)$ or $\lambda \in [0.175, 0.995]$).

In addition, it can result in an increase of the tax level in one country while decreasing it in the others ($\lambda = 1$ and $\gamma > 1.549$ or $\lambda \in [0, 0.175] \cup [0.995, 1)$ or $\lambda \in (0.175, 0.995)$ or $\gamma \in (\underline{\gamma}(\lambda), \bar{\gamma}(\lambda))$). The latter case cannot be captured by the symmetric equilibrium derived in Section 3.3 because it has the drawback of generating the same equilibrium environmental tax for all the countries. [8] These results show that omitting the heterogeneity of the industry sizes by countries when studying the effects of trade liberalization on the environmental taxes is likely to yield inaccurate outcomes.

3.4.3 Asymmetry and trade liberalization : the effects on welfare

Denote respectively by $SW_1(z)$ and $SW_2(z)$ the welfare of a country in the first and the second group of countries. What first clearly appears is that $SW_1(z)$, $SW_2(z)$ are second degree polynomials in z. In addition, they can be either concave or convex.

Indeed, plugging (3.17) and (3.18) into (3.7)-(3.12), one gets the equilibrium for the quantities $\bar{y}_i^p$ produced by each country. Since $\bar{y}_i^p$ is linear in z, the particular quadratic functional form of (3.15) in $\bar{y}_i^p$ shows that $SW_1(z)$ and $SW_2(z)$ are second degree polynomials in z. Moreover, for $k = 1, 2$, SW_k is concave if and only if $\frac{\partial^2 SW_k}{\partial z^2}(z) < 0$. It is convex when $\frac{\partial^2 SW_k}{\partial z^2}(z) > 0$.

In order to give a support to the above results, consider the case where $n_1 = 1, n_2 = 9$, $N_1 = 2$ and $N_2 = 1$, for arbitrary values of a, λ, γ and c. Whether SW_k is concave or convex depends on values of the spillover parameter and the damage cost parameter. Figure 3.1 and Figure 3.2 in Appendix illustrate these situations. For example, if $\lambda \in (0.4, 0.5)$ and $\gamma \in (0.15, 0.2)$ then $SW_1(z)$ and $SW_2(z)$ are both concave. If $\lambda \in (0, 0.5)$ and $\gamma \in (0, 0.05)$ then $SW_1(z)$ is convex. In the symmetric model of Section 3.3, we

8. Under the symmetric equilibrium assumption with identical countries, the equilibrium pollution tax is the same for each country. Therefore it is not possible for the equilibrium tax to increase in one country while it decreases in another as the effects of trade liberalization.

have proved that the social welfare is necessarily concave in the tariff z as in Baksi and Chaudhuri (2009). This simple example shows that such is not the case in the presence of asymmetry.

Proposition 13. *Let $\bar{x}_k$ be the solution of the equation $\frac{\partial SW_k}{\partial z}(z) = 0$ and $\bar{z}_k = \max(0, \bar{x}_k)$.*
(i) When SW_k is concave, we have $\frac{\partial SW_k}{\partial z}(z) < 0$ if and only if $z > \bar{z}_k$. Trade liberalization improves welfare in the typical country of group k only when the initial tariff is not too small.
(ii) When SW_k is convex, we have $\frac{\partial SW_k}{\partial z}(z) < 0$ if and only if $z < \bar{z}_k$. Trade liberalization increases welfare in each country of group k only when the initial tariff is not too large.

For the particular case where the pollution is purely local ($\lambda = 0$) and where $n_1 = 1, n_2 = 9$, $N_1 = 2$; $N_2 = 1$ and $\gamma = 1$, we get : $\frac{\partial SW_1}{\partial z}(z) \leq 0$ if and only if $z \geq 0.15(a-c)$. This result implies that $\frac{\partial SW_1}{\partial z}(z) > 0$, for all $z < 0.015(a-c)$. Hence, trade liberalization lowers welfare of all the countries in the first group when the initial tariff is lower than $0.015(a-c)$. Recall that in the symmetric framework, as in Baksi and Chaudhuri (2009) and Burguet and Sempere(2003), we have shown that trade liberalization always improves welfare when the pollution is purely local. This simple case highlights the limitation of that finding in the presence of asymmetry.

3.5 Conclusion

This paper has investigated the impacts of trade liberalization on equilibrium output, equilibrium pollution taxes and equilibrium welfare. Unlike the existing literature, we have considered the multilateral aspect of trade and have distinguished two types of countries according to the size of their industry, with the number of countries of each type potentially different. We have proved that asymmetry impacts significantly the outcomes. This has been done by comparing the outcomes derived in the asymmetric setting to those in the symmetric setting where all the countries are identical.

In the symmetric setting, our results are similar to those obtained in Baksi and Chaudhuri (2009). Trade liberalization always increases output. If the pollution is harmful enough, countries will raise their environmental protection in response to trade liberalization as shown in Proposition 11. Furthermore, as in Burguet and Sempere(2003), trade liberalization always increases welfare when we have to do with a local pollution problem.

However, when asymmetry exists, those strategic interactions change. In addition to the classical result of the symmetric model, trade liberalization may actually increase output only for the countries in one of the two groups, while decreasing output for the countries in the other group. Even if the pollution is harmful enough, it can be optimal to soften environmental policies in response to trade liberalization. Moreover, trade liberalization may not improve welfare even for strictly local pollution.

In this paper we have assumed an exogenous number of firms in each group of countries. In a symmetric setting with clean goods, Horstmann and Markusen (1992) have shown that allowing for endogenous firms location decisions might affect the analysis. This could be an avenue for new research. But our results still highlight clearly that asymmetry plays a crucial role in the outcome of trade liberalization.

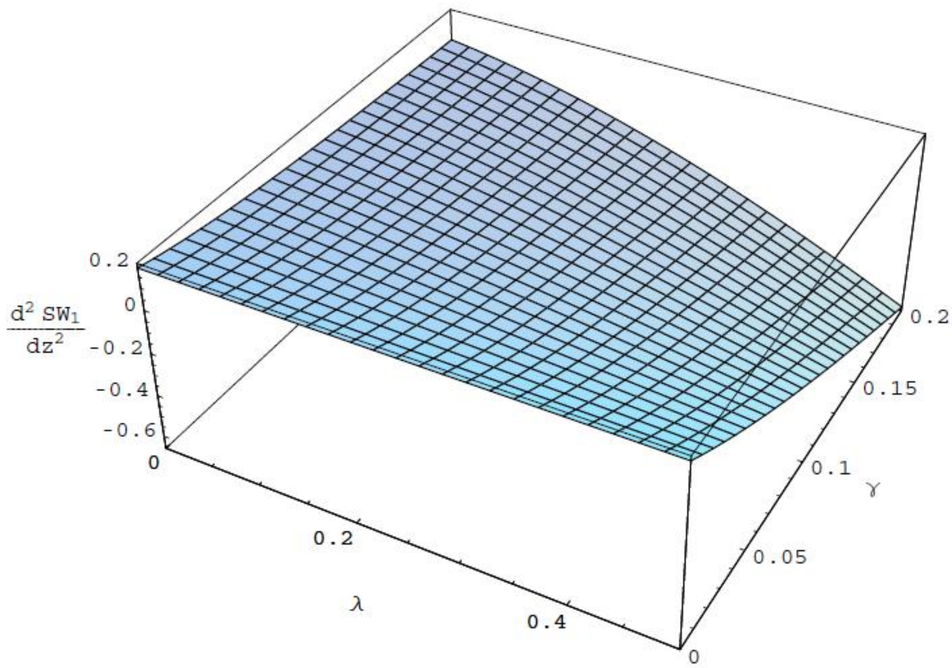

Figure 3.1 – Sign of $\frac{\partial^2 SW_1}{\partial z^2}$

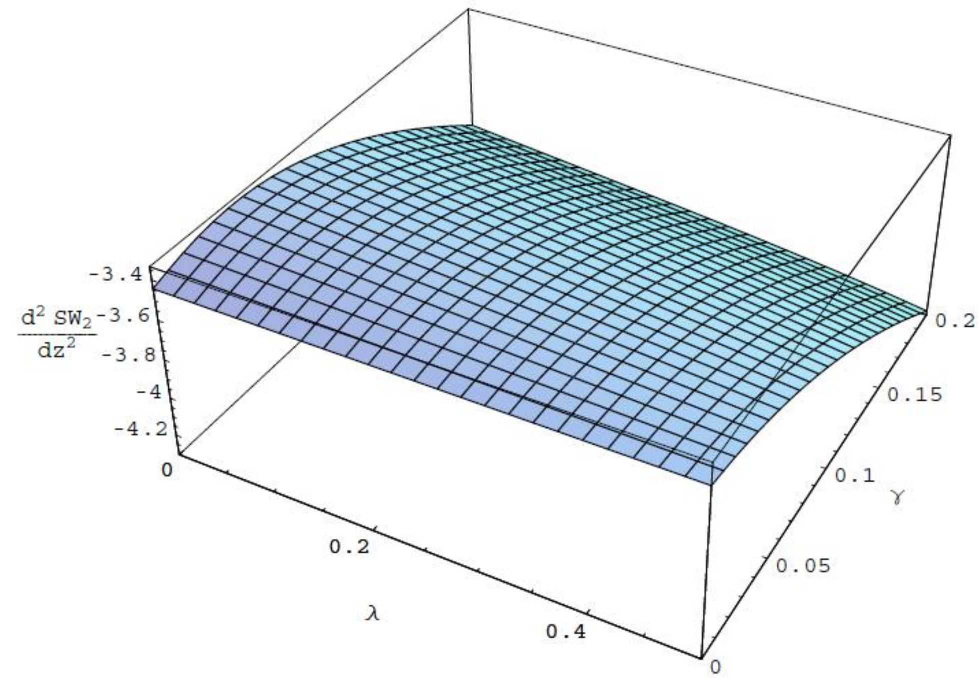

Figure 3.2 – Sign of $\frac{\partial^2 SW_2}{\partial z^2}$

CONCLUSION

Cette thèse est un recueil de trois articles analysant différentes politiques environnementales en présence de l'incertitude, du risque, du commerce international et de la pollution transfrontalière.

Dans le premier article, nous avons étendu le modèle de contrôle de la pollution de Dockner et Long (1993) sur deux aspects. Premièrement, un nombre arbitraire de pays est impliqué dans l'activité de pollution. Deuxièmement, à chaque instant, ces pollueurs font face à un risque d'un saut soudain de leurs dommages communs. Lorsque les pays agissent de façon coopérative, il s'avère que l'équilibre résultant est affecté de la même façon par la menace d'un saut des dommages que s'ils agissaient de façon non-coopérative. Le bien-être actualisé, le sentier des émissions et celui du stock de pollution sont plus faibles qu'en absence du risque. Une augmentation de ce risque réduit le bien-être actualisé et diminue le stock de pollution. Cependant, sur le long-terme, ce type d'incertitude peut avoir un effet positif sur le bien-être. Mais comme on pouvait s'y attendre, la politique unilatérale lègue un environnement de plus faible qualité et un plus faible niveau de bien-être que les stratégies prises en coopération.

Le second article de cette thèse a proposé un modèle de contrôle de la pollution par le canal des accords internationaux environnementaux. La littérature existante sur les accords internationaux dynamiques est bâtie sur l'une de deux approches. La première consiste à supposer que les stratégies d'adhésion et de pollution sont déterminées une fois pour toutes, en fonction du temps, au début de l'horizon d'étude de longueur infini. L'autre consiste à analyser le problème dans un modèle en temps discret, en supposant que les décisions d'adhésion et de pollution sont révisées au début de chaque période dont la longueur a été arbitrairement fixée à un. Ce second article a exploré la situation intermédiaire en traitant la longueur de la période d'engagement comme un paramètre positif et en étudiant les effets d'une variation de ce paramètre sur la taille des accords internationaux environnementaux stables. Nous avons montré que la durée de la période d'engagement peut avoir un impact très considérable sur la taille des accords internationaux environnementaux stables. Les résultats suggèrent que pour des durées de période d'engagement longues, les coalitions stables soutenables auront tendance à être de tailles relativement petites. Mais en dessous d'une valeur critique, plus on réduit la durée de la période d'engagement, plus la taille des coalitions stables tend à augmenter. Si la durée de la période d'engagement est suffisamment courte, le niveau de coopération sera le plus élevé. Puisque nos résultats dépendent des préférences très particulières et des simulations numériques, nous ne pouvons pas les prétendre universels. Mais ils montent clairement que la durée de la période d'engagement peut avoir des effets très significatifs sur l'issue des accords internationaux environnementaux. Ceci suggère d'accorder une attention particulière à la détermination de la durée d'engagement lors des discussions sur les accords internationaux de ce type. Pour les fins de notre analyse, il a été suffisant de traiter la durée de la période d'engagement comment un paramètre. Cependant, comment déterminer la durée de la période d'engagement optimale soulève une question qui mériterait d'être l'objet de recherches futures.

Le troisième article de cette thèse a analysé les impacts de la libéralisation du commerce sur la production d'équilibre, les taxes d'équilibre sur la pollution et le bien-être d'équilibre. Contrairement à la littérature existante, nous avons considéré l'aspect multilatéral du commerce et nous avons aussi distingué deux types de pays suivant la taille de leur industrie. Nous avons montré que l'asymétrie affecte significativement les résultats. Ceci a été fait en comparant les résultats trouvés dans le cadre asymétrique à ceux du contexte symétrique, où tous les pays sont identiques. Dans un cadre symétrique, nos résultats sont similaires à ceux trouvés par Baksi et Chaudhuri (2009). La libéralisation du commerce augmente la production. Si la pollution est suffisamment nuisible, les pays vont augmenter leur protection environnementale en réponse à la libéralisation du commerce. En outre, comme dans Burguet et Sempere (2003), la libéralisation du commerce augmente toujours le bien-être lorsque nous avons affaire à un problème de pollution locale. Cependant, lorsque l'asymétrie existe, ces interactions stratégiques changent. En plus du résultat classique des modèles symétriques, la libéralisation du commerce peut augmenter la production uniquement pour les pays dans un groupe et la réduire pour les pays dans l'autre groupe. Même si la pollution est très nuisible, il peut être optimal de diluer les politiques environnementales en réponse à la libéralisation du commerce. En plus, la libéralisation du commerce peut ne pas améliorer le bien-être, même si la pollution est strictement locale. Dans ce troisième article, nous avons considéré un nombre exogène de firmes dans chaque groupe de pays. Dans un contexte symétrique, avec des biens non polluant, Horstmann et Markusen (1992) ont montré que rendre endogène la décision du choix de localisation des firmes peut affecter les analyses. Ceci pourrait être une nouvelle avenue de recherche pour les politiques environnementales. Mais déjà nos résultats soulignent clairement que l'asymétrie joue un rôle crucial pour les impacts de la libéralisation du commerce.

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Annexe I

Appendix to Chapter 1

The following result known as the Gronwall's inequality, will be helpful in this appendix. Consider a function $y : [a,b] \to R$ which satisfies the following inequality : $\dot{y}(t) \geq uy(t) + v$ for all $t \in [a,b]$, with $y(a) = y_a$. We must have : $y(t) \geq e^{u(t-a)}[y_a + v\int_a^t e^{u(\tau-a)}d\tau]$, for all $t \in [a,b]$, where u, v and $b > a$ are arbitrary real numbers. For the proof, see for example Gronwall (1919).

I.1 Details for the Cooperative equilibrium

Consider first the state $\theta(t) = \theta + m$. The equilibrium value function is then obtained as the solution to the differential equation :

$$rW(z,\theta+m) = N\sigma^2/2 - N(\theta+m)z^2/2 + (N\sigma - \rho z)W_z(z,\theta+m) + NW_z(z,\theta+m)^2/2. \tag{I.1}$$

The quadratic structure of terms in the above equation suggests the following guess :

$$W(z,\theta+m) = -\frac{A}{2}z^2 - Bz + C,$$

from which we derive $W_z(z,\theta+m) = -Az - B$. Plugging those two expressions into (I.1) and equating the coefficients of powers of z, we get :

$$A = [-(2\rho+r) \pm \sqrt{(2\rho+r)^2 + 4N^2(m+\theta)}]/2N,$$
$$B = \sigma AN/[NA+r+\rho],$$
$$C = [\sigma^2 N - 2\sigma BN + B^2 N]/2r.$$

In order to assure the stability of the steady state we retain for A only the positive root :

$$A = [-(2\rho+r) + \sqrt{(2\rho+r)^2 + 4N^2(m+\theta)}]/2N > 0.$$

It is an easy matter to verify that :

$$A' \equiv \frac{\partial A}{\partial m} > 0; B' \equiv \frac{\partial B}{\partial m} = \frac{\partial B}{\partial A} \times \frac{\partial A}{\partial m} > 0; C' \equiv \frac{\partial C}{\partial m} = \frac{\partial C}{\partial B} \times \frac{\partial B}{\partial m} < 0.$$

We also have : $\sigma - B = \sigma(r+\rho)/(NA+r+\rho) > 0$, which implies that $\sigma > B$.

Consider now the state $\theta(t) = \theta$. The equilibrium value function is then obtained as the solution of :

$$(r+\beta)W(z,\theta) = N\sigma^2/2 - N\theta z^2/2 + (N\sigma - \rho z)W_z(z,\theta) + NW_z(z,\theta)^2/2 + \beta W^c(z,\theta+m). \tag{I.2}$$

Again, a plausible guess is :

$$W(z,\theta) = -\frac{a_1}{2}z^2 - a_2 z + a_3, \tag{I.3}$$

which yields $W_z(z,\theta) = -a_1 z - a_2$. Substituting into (I.2) and equating coefficients of powers of z of the resulting polynomials, we get :

$$\begin{aligned} a_1 &= [-(2\rho+r+\beta) \pm \sqrt{(2\rho+r+\beta)^2 + 4N(A\beta+N\theta)}]/2N, \\ a_2 &= \frac{N\sigma a_1 + \beta B}{Na_1 + \rho + r + \beta}, \\ a_3 &= [\sigma^2 N + 2C\beta - 2\sigma N a_2 + N a_2^2]/2(r+\beta). \end{aligned} \tag{I.4}$$

In order to assure the stability of the steady state, we retain only the positive root for a_1 :

$$a_1 = [-(2\rho+r+\beta) + \sqrt{(2\rho+r+\beta)^2 + 4N(A\beta+N\theta)}]/2N > 0.$$

We have, $\sigma - a_2 = [\sigma(r+\rho) + \beta(\sigma - B)]/(Na_1 + \rho + r + \beta) > 0$ which yields $\sigma > a_2$. There remains to show that $a_1' \equiv \partial a_1/\partial\beta > 0$, $a_2' \equiv \partial a_2/\partial\beta > 0$, and $a_3' \equiv \partial a_3/\partial\beta < 0$.

From (I.4), differentiating with respect β, we get :

$$a_1' = [-1 + ((2\rho+r+\beta) + 2AN)/\sqrt{(2\rho+r+\beta)^2 + 4N(A\beta+N\theta)}]/2N.$$

Hence $a_1' > 0$ if and only if $(2\rho + r + \beta) + 2AN > \sqrt{(2\rho + r + \beta)^2 + 4N(A\beta + N\theta)}$. Squaring both sides of this inequality, rearranging and using the fact that A satisfies the polynomial $NA^2 + (r + 2\rho)A = N(\theta + m)$, we obtain that the inequality is equivalent to $m > 0$, which is true if the apprehended jump in the damages is to be positive. Hence we can conclude that $a_1' > 0$.

$$a_2' = \{Na_1'[\sigma(r+\rho) + \beta(\sigma - B)] - Na_1(\sigma - B) + B(r+\rho)\}/(Na_1 + \rho + r + \beta)^2$$

First note that a_1 satisfies the polynomial $Na_1^2 + (r + \beta + 2\rho)a_1 - (A\beta + N\theta) = 0$, which, when differentiated with respect to β yields $2[Na_1 + (r + \beta + 2\rho)/2]a_1' = A - a_1$. The left-hand side being positive, we therefore have the $A - a_1 > 0$. Now using the fact that $B(r+\rho) = (\sigma - B)AN$, a_2' can be rewritten as $a_2' = \{Na_1'[\sigma(r+\rho) + \beta(\sigma - B)] + N(\sigma - B)(A - a_1)\}/(Na_1 + \rho + r + \beta)^2$, which is positive since $\sigma > B$, as just shown above.

$$a_3' = [N(B - a_2)(B - \sigma + a_2 - \sigma) + 2Na_2'(r + \beta)(a_2 - \sigma)]/2(r + \beta)^2,$$

because $2rC = N(\sigma - B)^2$. Since $\sigma > B$, $\sigma > a_2$, and
$B - a_2 = (A - a_1)(\sigma - B)N/(Na_1 + r + \rho + \beta) > 0$, it follows that $a_3' < 0$.

I.1.1 Proof of Proposition 2

(i) From (1.14a), we derive the following :

$$\frac{\partial z_L^c(t)}{\partial \beta} = -N[a_2'(\rho + Na_1) + Na_1'(\sigma - a_2)]\frac{1 - e^{-(\rho + Na_1)t}}{(\rho + Na_1)^2} - tNa_1'(z_0 - \frac{N(\sigma - a_2)}{\rho + Na_1})e^{-(\rho + Na_1)t}$$

If $z_0 \geq [N(\sigma - a_2)]/[\rho + Na_1]$, then $\partial z_L^c(t)/\partial \beta < 0$ for all $t \in (0, \nu]$, since $a_2' > 0$, $a_1' > 0$ and $\sigma - a_2 > 0$. If $z_0 < [N(\sigma - a_2)]/[\rho + Na_1]$, then $\partial z_L^c(t)/\partial \beta < 0$ if and only if :

$$tNa_1'(-z_0 + \frac{N(\sigma - a_2)}{\rho + Na_1})e^{-(\rho + Na_1)t} < N[a_2'(\rho + Na_1) + Na_1'(\sigma - a_2)]\frac{1 - e^{-(\rho + Na_1)t}}{(\rho + Na_1)^2}$$

Rearranging, one gets :

$$a_1'(-z_0+\frac{N(\sigma-a_2)}{\rho+Na_1})<\frac{[a_2'(\rho+Na_1)+Na_1'(\sigma-a_2)]}{(\rho+Na_1)}\frac{e^{(\rho+Na_1)t}-1}{(\rho+Na_1)t}. \tag{I.5}$$

Set $\psi(s)=\frac{e^s-1}{s}$ for all $s>0$. We have $\psi'(s)>0$ and $\psi(s)>1$ for all $s>0$. We also have $a_1'(-z_0+\frac{N(\sigma-a_2)}{\rho+Na_1})<\frac{a_2'(\rho+Na_1)+Na_1'(\sigma-a_2)}{(\rho+Na_1)}$ for non-negative values of z_0. Combining these two facts allows to see that inequality (I.5) always holds. Therefore $\partial z_L^c(t)/\partial\beta<0$ in that case as well. We can therefore conclude that $z_L^c(t)<z_L^c(t)_{|\beta=0}=\tilde{z}_c(t)$ for all $t\in(0,\nu]$.
(ii) Set $A^*=A_{|_{m=0}}$; $B^*=B_{|_{m=0}}$; $A^*=a_{1|_{\beta=0}}$ and $B^*=a_{2|_{\beta=0}}$, with a_1, a_2, A and B as given in Section 1.3, and set $y(t)=\tilde{q}_c(t)-q_L^c(t)=q_L^c(t)_{|_{\beta=0}}-q_L^c(t)$. We want to show that $y(t)>0$ for all $t\in[0,\nu)$. From (1.14a) and (1.14b), one obtains :

$$y(t)=a_2-B^*+a_1z_L^c(t)-A^*\tilde{z}_c(t), \tag{I.6}$$

and $y(0)=a_2-B^*+z_0(a_1-A^*)$. Since $a_2>B^*$ and $a_1>A^*$, it follows that $y(0)>0$. Differentiating equation I.6 with respect to time yields : $\dot{y}(t)=a_1\dot{z}_L^c(t)-A^*\dot{\tilde{z}}_c(t)$. Making use of (2.1), (1.14a) and (1.14b), this expression can be rewritten as $\dot{y}(t)=-(\rho+NA^*)y(t)+N(a_1-A^*)q_L^c(t)+\rho(a_2-B^*)$. Since $q_L^c(t)\geq 0$, $a_1>A^*$ and $a_2>B^*$, it follows that $\dot{y}(t)\geq-(\rho+NA^*)y(t)$ for all $t\in[0,\nu)$. By Gronwall's inequality, we then have $y(t)\geq y(0)e^{-(\rho+NA^*)t}>0$ for all $t\in[0,\nu)$.
(iii) Since $z_0\geq 0$, $a_1'>0$, $a_2'>0$ and $a_3'<0$, it follows that $W_L^c(0)=-a_1z_0^2/2-a_2z_0+a_3<W_L^c(0)_{|_{\beta=0}}\equiv\tilde{W}_c(0)$.

I.1.2 Proof of corollary 1

(i) Using result (i) from Proposition 2 at the instant of time $t=\nu$, one gets : $\tilde{z}_c(\nu)>z_\nu^c$. Set $g(t)=\tilde{z}_c(t)-z_H^c(t)$ for all $t\geq\nu$. Clearly we have $g(\nu)=\tilde{z}_c(\nu)-z_\nu^c>0$. It is easy to show that $\dot{g}(t)=-(\rho+NA^*)g(t)+N(B-B^*)+N(A-A^*)z_H^c(t)$. Since $A>A^*$, $B>B^*$ and $z_H^c(t)\geq 0$ we have $\dot{g}(t)\geq-(\rho+NA^*)g(t)$. Making use of Gronwall's inequality, we obtain $g(t)\geq g(\nu)e^{-(\rho+NA^*)(t-\nu)}>0$, for all $t\geq\nu$. Result (i) then follows.

(ii) Let us first prove that $q_H^c(\nu) = \sigma - B - Az_\nu^c < \tilde{q}_c(\nu) = \sigma - B^* - A^*\tilde{z}_c(\nu)$. Notice that $\lim_{\beta \to +\infty} a_1(\beta) = A$ and $\lim_{\beta \to +\infty} a_2(\beta) = B$; in addition, $a_1(\beta)$, $a_2(\beta)$ are increasing in β, and hence A and B are respectively their minimum upper bound. Since $q_L^c(t) = \sigma - a_2 - a_1 z_L^c(t) \leq \sigma - B^* - A^*\tilde{z}_c(t) = \tilde{q}_c(t)$ for all $t \in [0, \nu)$, by continuity of $\tilde{z}_c(t)$ and $z_L^c(t)$ at the point $t = \nu$ that inequality also works for $t = \nu$. Hence we have $\sigma + B + Az_\nu^c > \sigma + a_2(\beta) + a_1(\beta) z_\nu^c \geq \sigma + B^* + A^*\tilde{z}_c(\nu)$. Rearranging the first and the last term of these inequalities, one obtains $q_H^c(\nu) = \sigma - B - Az_\nu^c < \sigma - B^* - A^*\tilde{z}_c(\nu) = \tilde{q}_c(\nu)$.

Now, we are going to prove that $q_H^c(t) < \tilde{q}_c(t)$ for all $t \geq \nu$. Set $p(t) = \tilde{q}_c(t) - q_H^c(t)$ for all $t \geq \nu$. Since $q_H^c(\nu) < \tilde{q}_c(\nu)$, we have $p(\nu) > 0$. Using a similar method as above, we get that $\dot{p}(t) = -(\rho + NA^*)p(t) + \rho(B - B^*) + N(A - A^*)q_H^c(t)$, from which we derive : $\dot{p}(t) > -(\rho + NA^*)p(t)$ for all $t \geq \nu$. Applying Gronwall's inequality, we obtain $p(t) \geq p(\nu)e^{-(\rho + NA^*)(t-\nu)} > 0$ for all $t \geq \nu$. Hence $\tilde{q}_c(t) > q_H^c(t)$ for all $t \geq \nu$.

I.1.3 Proof that $W(z_c^{stea}) > \tilde{W}(\tilde{z}_c^{stea})$ if and only if $\rho(r+\rho)^2 \leq N^2\theta(r-\rho)$.

Recall that $\tilde{W}(\tilde{z}_c^{stea}) = W(z_c^{stea})_{|m=0}$ and $W(z_c^{stea}) \equiv W(z_c^{stea}, \theta + m) = -A(z_c^{stea})^2/2 - Bz_c^{stea} + C$. Replacing A, B, C and z_c^{stea} by their values given respectively in Section 1.3.1 and in Proposition 1, we get :

$$W(z_c^{stea}) = \sigma^2 N(r+\rho)[N^2(\rho - r)(\theta + m) + \rho^2(\rho + r)]/2r[\rho(r+\rho) + N^2(\theta + m)]^2,$$

from which we derive

$$\partial W(z_c^{stea})/\partial m = -\sigma^2 N^3(r+\rho)[N^2(\rho - r)(\theta + m) + \rho(\rho + r)^2]/2r[\rho(r+\rho) + N^2(\theta + m)]^3.$$

It is helpful to distinguish three cases.
Case 1 : if $\rho \geq r$ then, $\partial W(z_c^{stea})/\partial m < 0$ for all $m > 0$. Hence $W(z_c^{stea})_{|m=0} > W(z_c^{stea})$.
Case 2 : if $\rho(r+\rho)^2 > N^2\theta(r-\rho) > 0$, then $W(z_c^{stea})$ is convex in m ; in addition, we have $\lim_{m\to\infty} W(z_c^{stea}) = 0 < W(z_c^{stea})_{|m=0}$. Therefore $W(z_c^{stea})_{|m=0} > W(z_c^{stea})$.
Case 3 : if $N^2\theta(r-\rho) \geq \rho(r+\rho)^2$, $\partial W(z_c^{stea})/\partial m > 0$ for all $m > 0$. Hence $W(z_c^{stea}) >$

$W(z_c^{stea})_{|m=0}$. The result then follows.

I.2 Details for the non-cooperative equilibrium

Consider first the state $\theta(t) = \theta + m$. The equilibrium value function must then be a solution to the following differential equation :

$$rV(z,\theta+m) = (N-1/2)V_z(z,\theta+m)^2 + (N\sigma-\rho z)V_z(z,\theta+m) + \sigma^2/2 - (\theta+m)z^2/2. \tag{I.7}$$

Given the quadratic nature of the instantaneous benefit function, a plausible guess is :

$$V(z,\theta+m) = -\frac{\hat{A}}{2}z^2 - \hat{B}z + \hat{C}. \tag{I.8}$$

Using a similar argument as for the cooperative equilibrium, we get that this will indeed be a solution if :

$$\begin{aligned}
&\hat{A} = [-(2\rho+r) + \sqrt{(2\rho+r)^2 + 4(2N-1)(\theta+m)}]/2(2N-1) > 0,\\
&\hat{B} = \sigma N\hat{A}/[r+\rho+(2N-1)\hat{A}],\\
&\hat{C} = [\sigma^2 - 2\sigma N\hat{B} + (2N-1)\hat{B}^2]/2r.
\end{aligned} \tag{I.9}$$

We have $\sigma - \hat{B} = \sigma[(r+\rho)+\hat{A}(N-1)]/[r+\rho+(2N-1)\hat{A}] > 0$, and hence $\sigma > \hat{B}$. Applying the chain rule for differentiation, we obtain the following results :

$$\hat{A}' \equiv \frac{\partial \hat{A}}{\partial m} > 0; \hat{B}' \equiv \frac{\partial \hat{B}}{\partial m} = \frac{\partial \hat{B}}{\partial \hat{A}} \times \frac{\partial \hat{A}}{\partial m} > 0; \hat{C}' \equiv \frac{\partial \hat{C}}{\partial m} = \frac{\partial \hat{C}}{\partial \hat{B}} \times \frac{\partial \hat{B}}{\partial m} < 0.$$

Consider next the state $\theta(t) = \theta$. The equilibrium value function must then solve :

$$(N-1/2)V_z(z,\theta)^2 + (N\sigma-\rho z)V_z(z,\theta) - (r+\beta)V(z,\theta) + \beta V(z,\theta+m) + \sigma^2/2 - \theta z^2/2 = 0. \tag{I.10}$$

Again a plausible guess is :

$$V(z,\theta) = -\frac{u_1}{2}z^2 - u_2 z + u_3. \tag{I.11}$$

Using a similar method as for the cooperative equilibrium, we find that it will indeed be a solution if :

$$u_1 = [-(2\rho+r+\beta)+\sqrt{(2\rho+r+\beta)^2+4(2N-1)(\hat{A}\beta+\theta)}]/2(2N-1),$$
$$u_2 = \frac{N\sigma u_1+\beta\hat{B}}{(2N-1)u_1+\rho+r+\beta},$$
$$u_3 = [\sigma^2+2\beta\hat{C}-2\sigma N u_2+u_2^2(2N-1)]/2(r+\beta).$$

There remains to determine the signs of $u_1' \equiv \partial u_1/\partial\beta$, $u_2' \equiv \partial u_2/\partial\beta$, and $u_3' \equiv \partial u_3/\partial\beta$.

$$u_1' = [-1+\frac{(\beta+r+2\rho)+2(2N-1)\hat{A}}{\sqrt{(2\rho+r+\beta)^2+4(2N-1)(\hat{A}\beta+\theta)}}]/2(2N-1).$$

Using a similar argument as for the proof of $a_1' > 0$, while taking into account the fact that $\hat{A}$ satisfies $(2N-1)\hat{A}^2+(r+2\rho)\hat{A} = m+\theta$, we verify that $u_1' > 0$. Notice that u_1 satisfies the polynomial $(2N-1)u_1^2+(2\rho+\beta+r)u_1-(\hat{A}\beta+\theta) = 0$, which, upon differentiation with respect to β, yields $u_1'[2(2N-1)u_1+2\rho+\beta+r] = \hat{A}-u_1$. Since the left-hand side of this equality is positive, so is its right-hand side. Therefore $\hat{A} > u_1$ as stated.

$$u_2' = \{N\sigma(\rho+r)u_1'+\beta u_1'(N\sigma-(2N-1)\hat{B})+u_1[(2N-1)\hat{B}-N\sigma]$$
$$+\hat{B}(r+\rho)\}/[(2N-1)u_1+r+\rho+\beta]^2.$$

Since the denominator of u_2' is positive, its sign is that of its numerator. Using (I.9), the numerator of u_2' can be rewritten as : $N\sigma(\rho+r)u_1' + \frac{N\sigma(r+\rho)\beta}{r+\rho+(2N-1)\hat{A}}u_1' + \frac{N\sigma(r+\rho)}{r+\rho+(2N-1)\hat{A}}(\hat{A}-$

u_1), which is positive since each of the terms are positive. Therefore $u_2' > 0$.

$$u_3' = \{(\hat{B}-u_2)((2N-1)\hat{B}-\sigma N+(2N-1)u_2-\sigma N)+2(r+\beta)u_2'[(2N-1)u_2-\sigma N]\}/2(r+\beta)^2,$$

because $2r\hat{C} = \sigma^2 - 2\sigma N\hat{B} + (2N-1)\hat{B}^2$. Since we have :
$(2N-1)\hat{B} - \sigma N = -\sigma N(r+\rho)/[(2N-1)\hat{A}+r+\rho] < 0$,
$(2N-1)u_2 - \sigma N = [-\sigma N(r+\rho)+\beta((2N-1)\hat{B}-\sigma N)]/[(2N-1)u_1+r+\rho+\beta] < 0$,
and
$\hat{B}-u_2 = -\sigma N(r+\rho)(2N-1)(u_1-\hat{A})/[(2N-1)u_1+r+\rho+\beta][r+\rho+(2N-1)\hat{A}] > 0$,
it follows that $u_3' < 0$.

I.2.1 Proof that $A > \hat{A}$, $B > \hat{B}$, $a_1 > u_1$ and $a_2 > u_2$

Since $N^2 > 2N-1$ for $N \geq 2$, we have :

$$\begin{aligned} -(2\rho+r)+\sqrt{(2\rho+r)^2+4N^2(\theta+m)} &> -(2\rho+r)+\sqrt{(2\rho+r)^2+4(2N-1)(\theta+m)} \\ 1/2N &\geq 1/2(2N-1) \end{aligned}$$

Multiplying side by side both inequalities, we verify that $A > \hat{A}$.

We have $B - \hat{B} = \sigma N[(r+\rho)(A-\hat{A})+(N-1)A\hat{A}]/(r+\rho+NA)(r+\rho+(2N-1)\hat{A}) > 0$ and hence $B > \hat{B}$.

Since $4N(A\beta+N\theta) > 4(2N-1)(\hat{A}\beta+\theta)$ for $N \geq 2$, it follows that :

$$\begin{aligned} &-(2\rho+r+\beta)+\sqrt{(2\rho+r+\beta)^2+4N(A\beta+N\theta)} > \\ &\quad -(2\rho+r+\beta)+\sqrt{(2\rho+r+\beta)^2+4(2N-1)(\hat{A}\beta+\theta)} \end{aligned}$$

But recall that we also have :

$$1/2N > 1/2(2N-1).$$

By multiplying side by side those two inequalities we verify that $a_1 > u_1$.

Finally, since $B > \hat{B}$, using (I.4) it is easy to verify that :

$$a_2 > \frac{N\sigma a_1 + \beta\hat{B}}{(2N-1)a_1 + \rho + r + \beta} = f(a_1),$$

from which we derive :

$$f(a_1) - f(u_1) = \frac{(a_1 - u_1)[\sigma N(\rho + r) + \sigma N\beta - (2N-1)\beta\hat{B}]}{((2N-1)a_1 + \rho + \beta + r)((2N-1)u_1 + \rho + \beta + r)}.$$

Using (I.9), we get that $\sigma N\beta - (2N-1)\beta\hat{B} = N\sigma\beta(r+\rho)/(r+\rho+(2N-1)\hat{A}) > 0$. It then follows that $a_2 > f(a_1) > f(u_1) = u_2$.

I.2.2 Proof of Proposition 5

(i) From Proposition 1 we know that $pr(0 < \nu < \infty) = 1$. Thus, almost surely the state of high damages must occur at a finite date. This means that time path of the stock of pollutant is in the long run given by $z_H^n(t)$, which converges to z_n^{stea}. Since $A > \hat{A}$ and $B > \hat{B}$ we have :

$$\begin{aligned} N(\sigma - \hat{B}) &> N(\sigma - B) \\ 1/(\rho + N\hat{A}) &> 1/(\rho + NA). \end{aligned}$$

Multiplying those inequalities side by side, we get :

$$z_n^{stea} = N(\sigma - \hat{B})/(\rho + N\hat{A}) > N(\sigma - B)/(\rho + NA) = z_c^{stea}.$$

The proof that the presence of the risk of a jump in the damages results in a lower stock of pollutant than when that risk is not present is similar to that used to derive the same result for the cooperative equilibrium.

(ii) Notice that at the steady state, we have : $\dot{z} = Nq - \rho z = 0$. Hence, $q_n^{stea} = \rho z_n^{stea}/N$ and $q_c^{stea} = \rho z_c^{stea}/N$. Since we have just shown that $z_n^{stea} > z_c^{stea}$, it follows that $q_n^{stea} > q_c^{stea}$.

I.2.3 Proof of Proposition 6

(i) Using a similar method as for the proof of (ii) in Proposition 2, with u_2 replacing a_2 and u_1 replacing a_1 we get $\tilde{q}_n(t) \equiv q_L^n(t)_{|\beta=0} > q_L^n(t)$ for all $t \in (0,\nu]$. By a similar argument as for the proof of (ii) in Corollary 1, we obtain $\tilde{q}_n(t) \equiv q_H^n(t)_{|m=0} > q_H^n(t)$ for all $t \in (\nu,\infty]$.
(ii) The solution for $z_L^n(t)$, for $t \in (0,\nu]$, can be obtained from $z_L^c(t)$ by replacing a_1 by u_1 and a_2 by u_2. The proof of $\partial z_L^c(t)/\partial\beta < 0$ for all $t \in (0,\nu]$ in Proposition 2 rested only on the facts that $a_1, a_1', a_2, a_2' > 0$. Since $u_1, u_1', u_2, u_2' > 0$, we can conclude that $\partial z_L^n(t)/\partial\beta < 0$ for all $0 < t \leq \nu$ as well. Therefore $\tilde{z}_n(t) \equiv z_L^n(t)_{|\beta=0} > z_L^n(t)$ for all $t \in (0,\nu]$. We also have $\tilde{z}_n(t) \equiv z_H^n(t)_{|m=0} > z_H^n(t)$ for all $t \in (\nu,\infty]$. Indeed, its proof is similar to that of (i) in Corollary 1 in which $\hat{A}^* \equiv \hat{A}_{|m=0}$ plays the role of A^* whereas $\hat{B}^* \equiv \hat{B}_{|m=0}$ plays the role of B^*.
(iii) Since $z_0 \geq 0$, $u_1' > 0$, $u_2' > 0$ and $u_3' < 0$, we have $V_L^n(0) = -u_1 z_0^2/2 - u_2 z_0 + u_3 < V_L^n(0)_{|\beta=0} \equiv \tilde{V}_n(0)$.

I.3 Comparison of the cooperative and non-cooperative equilibria

Set $\Delta(t) = q_L^n(t) - q_L^c(t)$ for all $t \in [0,\nu)$. Using the expressions for $q_L^c(t)$ and $q_L^n(t)$ given respectively in Section 1.3 and in Section 1.4, we get $\Delta(t) = a_1 z_L^c(t) - u_1 z_L^n(t) + a_2 - u_2$. Since $z_0 \geq 0, a_1 > u_1$ and $a_2 > u_2$, it follows that $\Delta(0) = (a_1 - u_1)z_0 + a_2 - u_2 > 0$. Differentiating $\Delta(t)$, we obtain $\dot{\Delta}(t) = N(a_1 - u_1)q_L^c(t) - (\rho + Nu_1)\Delta(t) + \rho(a_2 - u_2)$. Since $q_L^c(t) > 0$, $a_1 > u_1$ and $a_2 > u_2$, it follows that $\dot{\Delta}(t) \geq -(\rho + Nu_1)\Delta(t)$ for all $t \in [0,\nu)$. Using Gronwall's inequality, we get $\Delta(t) \geq \Delta(0)e^{-(\rho+Nu_1)t} > 0$. Thus $q_L^n(t) > q_L^c(t)$ for all $t \in [0,\nu)$. Applying again Gronwall's inequality, we verify that $q_H^n(t) > q_H^c(t)$ for all $t \geq \nu$.

Now set $h(t) = z_L^n(t) - z_L^c(t)$ for all $t \in [0,\nu]$. We have $h(0) = z_0 - z_0 = 0$ and $\dot{h}(t) = N(a_2 - u_2) - (\rho + Nu_1)h(t) + N(a_1 - u_1)z_L^c(t)$. Since $a_2 > u_2$, $a_1 > u_1$, and $z_L^c(t) \geq 0$, it follows that $\dot{h}(t) \geq -(\rho + Nu_1)h(t) + N(a_2 - u_2)$. Using Gronwall's inequality, we get $h(t) > h(0)e^{-(\rho+Nu_1)t} = 0$ for all $t \in (0,\nu]$. Hence $z_L^n(t) > z_L^c(t)$ for all $t \in (0,\nu]$. Using once more Gronwall's inequality, we verify similarly that $z_H^n(t) > z_H^c(t)$ for all $t \geq \nu$.

Finally, we may compare the steady-state levels of welfare. In the two cases, the steady state occurs after the state $\theta + m$ is reached. Set $\mu(z) = W(z, \theta + m)/N$ and $V(z) = V(z, \theta + m)$, the welfare of each individual country in respectively the cooperative and the non-cooperative equilibrium. By definition the cooperative solution maximizes the global welfare of the N identical countries and hence, for any given identical initial stock of pollution z, $\mu(z) \geq V(z)$. In particular we have $\mu(z_c^{stea}) \geq V(z_c^{stea})$. But we know from Proposition 5 that $z_n^{stea} > z_c^{stea}$, therefore, since clearly $V_z(z) < 0$, we have $\mu(z_c^{stea}) > V(z_n^{stea})$.

Annexe II

Appendix to Chapter 2

II.1 The fully-cooperative equilibrium

We use the quadratic guess $V_i(z) = -\bar{A}z^2/2N - \bar{B}z/N + \bar{C}/N$. Using (2.11)-(2.13) for $\Psi = V_i$, $n = N$ and for $q_i \in (0,1)$ we get :

$$q_i(z_k) = \frac{af(r,h) - \bar{B}Nf(\rho,h)e^{-rh} - Nz_k(\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)})}{N^2[\lambda_1 + \bar{A}f(\rho,h)^2 e^{-rh}]}. \tag{II.1}$$

Substituting (II.1) into the Bellman equation and using the envelope theorem, one obtains :

$$V_i'(z) = -z\gamma f(r+2\rho,h) - N\lambda_2 q_i(z) + e^{-h(r+\rho)} V_i'(Nf(\rho,h)q_i(z) + ze^{-\rho h}), \forall z \geq 0.$$

Equating the coefficients of the *LHS* and *RHS* of this first-degree polynomial in z, we find that

$$\bar{A} = \left[-\bar{a}_1 + \sqrt{\bar{a}_1^2 - 4\bar{a}_2\bar{a}_0}\right]/2\bar{a}_2,$$

which is the positive root of the second degree polynomial $\bar{a}_2 A^2 + \bar{a}_1 A + \bar{a}_0 = 0$, where,

$$\begin{aligned}
&\bar{a}_2 = f(\rho,h)^2 e^{-rh}, \\
&\bar{a}_1 = \lambda_1(1 - e^{-h(r+2\rho)}) + \lambda_2(1+N)f(\rho,h)e^{-h(r+\rho)} - N\gamma f(r+2\rho,h)f(\rho,h)^2 e^{-rh}, \\
&\bar{a}_0 = N[\lambda_2^2 - \gamma\lambda_1 f(r+2\rho,h)].
\end{aligned}$$

Given $\bar{A}$, we find :

$$\bar{B} = \frac{af(r,h)(N\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)})}{N(\lambda_1 + \bar{A}f(\rho,h)^2 e^{-rh})(1 - e^{-h(r+\rho)}) + Nf(\rho,h)e^{-rh}(N\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)})}.$$

Given $\bar{A}$ and $\bar{B}$, we get
$\bar{C} = N\bar{\beta}[af(r,h) - \bar{B}f(\rho,h)e^{-rh}]/(1-e^{-rh}) - N^2\bar{\beta}^2[N\lambda_1 + \bar{A}f(\rho,h)^2e^{-rh}]/2(1-e^{-rh})$,
where $\bar{\beta} = [af(r,h) - N\bar{B}f(\rho,h)e^{-rh}]/N^2[\lambda_1 + \bar{A}f(\rho,h)^2e^{-rh}]$.
The dynamic evolution of the stock of pollutant is then

$$\begin{aligned} z_{k+1} &= f(\rho,h)\frac{af(r,h) - \bar{B}Nf(\rho,h)e^{-rh} - z_k N(\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)})}{N\lambda_1 + N\bar{A}f(\rho,h)^2e^{-rh}} + z_k e^{-\rho h} \\ &\equiv \varphi_N(z_k). \end{aligned}$$

The unique solution to the equation $x = \varphi_N(x)$ is :

$$\bar{z} = \frac{af(r,h) - \bar{B}Ne^{-rh}f(\rho,h)}{N(\lambda_2 + \bar{A}f(\rho,h)e^{-rh}) + \rho N[\lambda_1 + \bar{A}f(\rho,h)^2e^{-rh}]}.$$

Hence, $\forall\ k = 0,1,2,3,...,$

$$z_{k+1} - \bar{z} \equiv \varphi_N(z_k) - \bar{z} = R_N(z_k - \bar{z})$$

so that

$$z_k = \bar{z} + (R_N)^k(z_0 - \bar{z}), \quad \forall \quad k = 0,1,2,3,..., \tag{II.2}$$

where,

$$R_N = \frac{\lambda_1 e^{-\rho h} - \lambda_2 f(\rho,h)}{\lambda_1 + \bar{A}f(\rho,h)^2e^{-rh}}.$$

We have $1 > R_N$. Indeed, because all the parameters are non-negative and $\bar{A} > 0$, we have the following inequality :

$$(1-e^{-\rho h})\lambda_1 + \bar{A}f(\rho,h)^2e^{-rh} + \lambda_2 f(\rho,h) > 0.$$

Rearranging the terms of this inequality, one gets :

$$\lambda_1 + \bar{A}f(\rho,h)^2e^{-rh} > \lambda_1 e^{-\rho h} - \lambda_2 f(\rho,h).$$

Dividing the LHS and the RHS of the last inequality by its LHS, one obtains $1 > R_N$. The sequence $z_k - \bar{z}$ being a geometric progression, a necessary and sufficient condition for z_k to converge is $1 > R_N > -1$. It has been establish that $1 > R_N$. Therefore, if and only if $R_N > -1$, z_k converges and its limit is $\bar{z}$. It converges monotonically if $R_N > 0$. The steady-state emission rate exists if and only $R_N > -1$ and is given by :

$$\bar{q}_i = \frac{af(r,h) - N\bar{B}f(\rho,h)e^{-rh} - \bar{z}N[\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)}]}{N^2[\lambda_1 + \bar{A}f(\rho,h)^2 e^{-rh}]}.$$

Notice that we have $0 < q_i(z_k) < 1$. Indeed, let us assume that inequalities from (2.17) hold. On the one hand, we must have
$af(r,h) < N^2[\lambda_1 + \bar{A}e^{-rh}f(\rho,h)^2] + N\bar{B}f(\rho,h)e^{-rh} + z_k N[\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)}]$ which, rearranging, yields $af(r,h) - N\bar{B}f(\rho,h)e^{-rh} - z_k N[\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)}] < N^2[\lambda_1 + \bar{A}e^{-rh}f(\rho,h)^2]$. Dividing the two sides of this inequality by its RHS, one gets $q_i(z_k) < 1$. On the other hand, it is easy to show that
$af(r,h) - N\bar{B}f(\rho,h)e^{-rh} \geq N[\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)}]\bar{z}$. Since from (II.2) we have $\bar{z} > z_{2k} \geq 0$, it then follows that $af(r,h) - N\bar{B}f(\rho,h)e^{-rh} > N[\lambda_2 + \bar{A}f(\rho,h)e^{-h(r+\rho)}]z_{2k}$. Rearranging this inequality, we can see that the numerator of (II.1) is positive at the point z_{2k}. Since the denominator of (II.1) is always positive, we can conclude that $q_i(z_{2k}) > 0$. Unfortunately, we have not been able to prove analytically that $q_i(z_{2k+1}) > 0$, $k = 0,1,2,....$ However, we take into account this constraint in our simulations.

II.2 Proof of proposition 10

Making use of (2.10) and the quadratic approximation $\Psi(z) = -\frac{A}{2}z^2 - Bz + C$, for all z, non-signatories will emit one at each period if and only if

$$af(r,h) \geq \lambda_1 Q + z_k \lambda_2 + f(\rho,h)e^{-rh}\{B + A[f(\rho,h)Q + z_k e^{-\rho h}]\} \tag{II.3}$$

Notice that, $Q \leq N$; $z_k \leq N/\rho$ and $f(\rho,h)Q + z_k e^{-\rho h} \leq N/\rho$. Replacing Q, z_k and $f(\rho,h)Q + z_k e^{-\rho h}$ by their upper bound in (II.3) yields

$$af(r,h) \geq N(\lambda_1 + \lambda_2/\rho) + f(\rho,h)e^{-rh}(B + AN/\rho), \tag{II.4}$$

which is condition (2.19). Thus if (2.19) holds, it will be optimal for a non-signatory to play $q_j = 1$ at each period.

Given the quadratic approximation of Ψ, for $q_j = 1$ relations (2.11)-(2.13) can be rewritten as

$$af(r,h) \leq -n^2[\lambda_1 + Ae^{-rh}f(\rho,h)^2] + n\tau(z_k,h), q_i = 0 \tag{II.5}$$

$$af(r,h) = n^2 q_i[\lambda_1 + Ae^{-rh}f(\rho,h)^2] - n^2[\lambda_1 + Ae^{-rh}f(\rho,h)^2] + n\tau(z_k,h), 0 \leq q_i \leq 1 \tag{II.6}$$

$$af(r,h) \geq n\tau(z_k,h), q_i = 1. \tag{II.7}$$

Solving (II.7), we get : $q_i(n,z_k) = 1$ if and only if $1 \leq n \leq n_0(z_k) \equiv af(r,h)/\tau(z_k,h)$.

Pick $\bar{n}_1(z_k)$ and $\bar{n}_2(z_k)$ to be the two roots of the second degree equation in n : $-n^2[\lambda_1 + Ae^{-rh}f(\rho,h)^2] + n\tau(z_k,h) = af(r,h)$. Their expressions are given by (2.21). Solving (II.5) yields $q_i(n,z_k) = 0$ if and and only if we have $\bar{n}_1(z_k) \leq n \leq \bar{n}_2(z_k)$.

Solving (II.6), we obtain

$$0 \leq q_i(n,z_k) = \frac{af(r,h) + n^2(\lambda_1 + Ae^{-rh}f(\rho,h)^2) - n\tau(z_k,h)}{n^2(\lambda_1 + Ae^{-rh}f(\rho,h)^2)} \leq 1, \tag{II.8}$$

if and only if $n \in [\bar{n}_0(z_k), \bar{n}_1(z_k)]$.

As illustrated in Figure II.1, $\bar{n}_1(z_k)$ and $\bar{n}_2(z_k)$ are two positive real numbers but only if there exists $n \in (0,N)$ for which : $-n^2[\lambda_1 + Ae^{-rh}f(\rho,h)^2] + n\tau(z_k,h)$ lies above $af(r,h)$. In addition, $\bar{n}_1(z_k) < N < \bar{n}_2(z_k)$ is satisfied only when we have the following :

$$-N^2[\lambda_1 + Ae^{-rh}f(\rho,h)^2] + N\tau(z_k,h) > af(r,h). \tag{II.9}$$

Notice that (i) $\tau(z,h)$ is increasing in z ; (ii) $z_0 < N/\rho$, implies that $\tau(z_k,h) \geq \tau(z_0,h)$,

$k = 0,1,2...$[1] (iii) If we replace z_k by z_0 in (II.9), we will get exactly the condition in (2.20). Thus if the condition $-N^2[\lambda_1 + Ae^{-rh}f(\rho,h)^2] + N\tau(z_0,h) > af(r,h)$ holds, it will be so for (II.9). The inequality (2.20) is then a sufficient condition to have $\bar{n}_1(z_k) < N < \bar{n}_2(z_k)$, $k = 0,1,2...$

Since we have : $n\tau(z_k,h) \geq -n^2[\lambda_1 + Ae^{-rh}f(\rho,h)^2] + n\tau(z_k,h)$, when conditions (2.19)-(2.20) hold, we also have $\bar{n}_0(z_k) \leq \bar{n}_1(z_k)$, as illustrated in Figure II.1. Finally, making use of $Q \leq N$; $z_k \leq \tilde{z}$ for all k, we have shown that $\bar{n}_0(z_k) \geq 1$ when the condition (2.20) is satisfied.

Summarizing, if conditions (2.19)-(2.20) are satisfied, we must have two results. On the one hand, we must have : $1 \leq \bar{n}_0(z_k) < \bar{n}_1(z_k) < N < \bar{n}_2(z_k)$. On the other hand, non-signatories must emit one at each period, while the decision rule of emission by signatories will be given by (2.23).

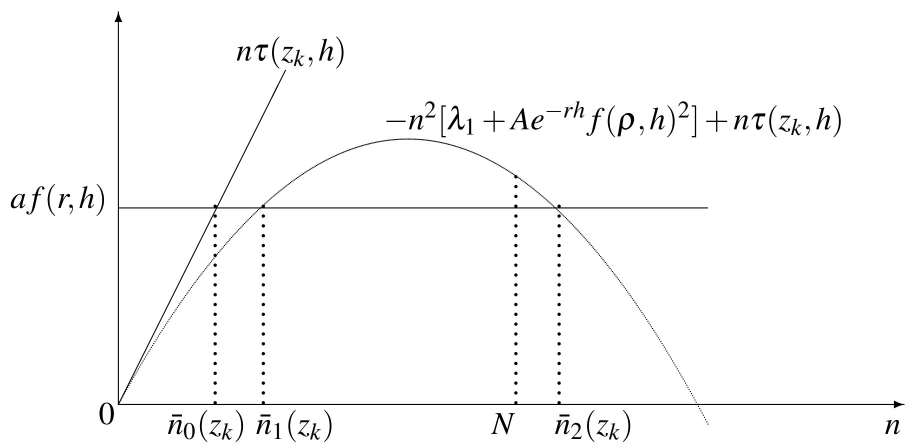

Figure II.1 – Critical values for n.

II.3 The algorithm

This section presents the algorithm used to approximate the value function Ψ. It is made up of the eight following steps.

(1) Generate values of h, using the formula, $h_p = h_{p-1} + 0.1$; $p = 1,...,4995$, with $h_0 =$

1. Our simulations show that for z_0 lying below the steady-state of the non-cooperative equilibrium : N/ρ, the stock of pollution increases over time so that its lower bound is z_0.

0.1. We choose plausible parameters a, γ, r, ρ, z_0 and N such that the conditions (2.16), (2.17), (2.19) and (2.20) always hold for all $h_p, p = 0, ..., 4995$.

(2) For each vector $(a, \gamma, r, \rho, z_0, N, h_p)$ of parameters we consider as initial values for A, B and C the values of the quadratic value function for the non-cooperative equilibrium.

(3) We generate 251 values of the stocks z in the interval $[z_0, \tilde{z}]$ following the sequence : $z_p = z_0 + p(\tilde{z} - z_0)/250$; $p = 0, ..., 250$, where $\tilde{z}$ represents the steady-state of the stock of pollutant for the non-cooperative equilibrium.

(4) For each value of z from (3), compute $\bar{n}_1(z)$ and $\bar{n}_2(z)$ using (2.21), $\bar{n}_0(z)$ using (2.22), and $q_i(n,z)$ using (2.23). Using (2.14) and (2.15), compute $\Lambda(n,z) = V_i(n,z) - V_j(n-1,z)$, for $\Psi(z) = -Az^2/2 - Bz + C$.

(5) For each value of z we calculate $n^*(z)$, the highest value of n for which $\Lambda(n,z) \geq 0$.

(6) We then compute for each of the 251 values of z, $n^*(z)$ and $q_i(n^*(z), z)$. We estimate the relation $Q(n^*(z), z) \approx \beta + \alpha z$ by linear regression which yields an estimation of (β, α).

(7) Replacing $Q(n^*(z), z)$ by $\beta + \alpha z$ in (2.18) and equating the coefficient of power of z, we get the new estimation of A, B and C, which are the following :

$$
\begin{aligned}
A &= [\alpha^2 \lambda_1 + 2\alpha\lambda_2 + \gamma f(r + 2\rho, h)]/[1 - e^{-rh}(e^{-\rho h} + \alpha f(\rho, h))^2], \\
B &= \frac{N\beta(\alpha\lambda_1 + \lambda_2) - a\alpha f(r,h) + NA\beta e^{-rh} f(\rho,h)(e^{-\rho h} + \alpha f(\rho,h))}{N[1 - e^{-rh}(e^{-\rho h} + \alpha f(\rho,h))]}, \\
C &= \frac{2a\beta f(r,h) - N\beta^2\lambda_1 - N\beta e^{-rh} f(\rho,h)(2B + A\beta f(\rho,h))}{2N(1 - e^{-rh})}.
\end{aligned}
$$

(8) We repeat this operation until convergence is obtained i.e., variations of A, B and C not greater than 5% or the number of iteration not greater than 101.

Annexe III

Appendix to Chapter 3

The first-order conditions for the pollution taxes are :

$$\frac{\partial SW_j}{\partial t_j}(t_j, t_{-j}) = 0 \quad \text{for all} \quad j \in N_1 \cup N_2. \tag{III.1}$$

For a symmetric equilibrium in each group, we have : $t_1 = t_j \; \forall j \in N_1$ and $t_2 = t_k \; \forall k \in N_2$. So, (III.1) can be rewritten as :

$$e_1 t_1 + e_2 t_2 \quad = v_1 z + v_2 (a - c), \tag{III.2}$$

$$\hat{e}_1 t_1 + \hat{e}_2 t_2 \quad = \hat{v}_1 z + \hat{v}_2 (a - c), \tag{III.3}$$

where

$e_1 = n_1^2 N_1 (1 + 2n_1 N)/d^2 - N n_1^2 (1 + N_1)/d + N B_1 n_1 [1 - \lambda + \lambda N_1 + (1 - \lambda) n_2 N_2]/d;$

$e_2 = n_1 n_2 N_2 (1 + 2n_1 N)/d^2 - n_1 n_2 N_2 N/d + N B_1 n_2 N_2 [\lambda - n_1 (1 - \lambda)]/d;$

$\hat{e}_1 = n_1 n_2 N_1 (1 + 2n_2 N)/d^2 - n_1 n_2 N_1 N/d + N B_2 n_1 N_1 [\lambda - n_2 (1 - \lambda)]/d;$

$\hat{e}_2 = n_2^2 N_2 (1 + 2n_2 N)/d^2 - N n_2^2 (1 + N_2)/d + N B_2 n_2 [1 - \lambda + \lambda N_2 + (1 - \lambda) n_1 N_1]/d;$

$v_1 = n_1 [1 + n_1 (2N - 1)]/d^2 + n_1 (1 + n_1 - N - d)/d + (N - 1) B_1 [(1 - \lambda) n_1 + \lambda (n_1 N_1 + n_2 N_2)]/d;$

$\hat{v}_1 = n_2 [1 + n_2 (2N - 1)]/d^2 + n_2 (1 + n_2 - N - d)/d + (N - 1) B_2 [(1 - \lambda) n_2 + \lambda (n_1 N_1 + n_2 N_2)]/d;$

$v_2 = -n_1 (1 + 2n_1 N)/d^2 + n_1 (1 + N)/d + N B_1 [(1 - \lambda) n_1 + \lambda (n_1 N_1 + n_2 N_2)]/d;$

$\hat{v}_2 = -n_2 (1 + 2n_2 N)/d^2 + n_2 (1 + N)/d + N B_2 [(1 - \lambda) n_2 + \lambda (n_1 N_1 + n_2 N_2)]/d;$

$B_1 = \gamma N n_1 [-1 + (1 - \lambda)(n_1 - n_1 N_1 - n_2 N_2)]/d;$

$B_2 = \gamma N n_2 [-1 + (1 - \lambda)(n_2 - n_1 N_1 - n_2 N_2)]/d;$

$d = 1 + n_1 N_1 + n_2 N_2.$

Solving (III.2) and (III.3), we get :

$$t_1 = [z(v_1\hat{e}_2 - e_2\hat{v}_1) + (a-c)(v_2\hat{e}_2 - e_2\hat{v}_2)]/(e_1\hat{e}_2 - e_2\hat{e}_1),$$
$$t_2 = [z(\hat{v}_1 e_1 - \hat{e}_1 v_1) + (a-c)(\hat{v}_2 e_1 - \hat{e}_1 v_2)]/(e_1\hat{e}_2 - e_2\hat{e}_1).$$